Test 1 Math 3336 September 3. 2019

Each problem is worth 20 points! You have the full class period to complete the test.

1. True or False.

The meaning of the implication $p \to q$ is

a. p only if q. T b. q if p. T c. p is sufficient for q T d. q is necessary for p T

2. Determine whether each of these conditional statements is true or false.

a. If $1+1=2,$ then $2+2=5.$ F

b. If $1+1=3,$ then $2+2=4$ T

c. If $1+1=3,$ then $2+2=5$ T

d. If $2+2=4,$ then $1+2=3$ T

3.Construct a truth table for each of these compound propositions.

a. $p \to (\neg q \vee r)$

p	q	r	$\neg q$	$\neg q \vee r$	p$\to (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
F	T	T	F	T	T
T	F	F	T	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

b. $\neg p \to (q \to r)$

c. $(p \to q) \vee (\neg p \to r)$

d. $(p \to q) \wedge (\neg p \to r)$

e. $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$

f.$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

4. Show that each of these conditional statements is a tautology by using truth tables.

a.

$[\neg p \wedge (p \vee q)] \to q$

p	q	$\neg p$	p$\vee q$	$\neg p \wedge (p \vee q)$	$\neg p \wedge (p \vee q) \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

b. $[(p \to q) \wedge (q \to r)] \to (p \to r)$

c. $[p \wedge (p \to q)] \to q$

d. $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

5. Show that $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$ are not logically equivalent. Answer: LHS is T where $p=T, q=F, r=F$ while RHS F

6. Find a compound proposition in propositional variables $p, q,$ and r that is true when

exactly two of $p, q,$ and r are true and is false otherwise. Answer:
$(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$

7. Find the disjunctive and conjunctive normal form for $f(p, q)$ if it has the following truth table:

p	q	$f(p,q)$
T	T	F
T	F	T
F	T	T
F	F	F

$f(p,q) \equiv (p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q); \neg f(p,q) \equiv p \wedge q, f(p,q) \equiv \neg p \vee \neg q$

8. Show that $(p \to q) \equiv (\neg q \to \neg p)$ $(p \to q) \equiv \neg p \vee q \equiv \neg(\neg q) \vee \neg p \equiv (\neg q \to \neg p)$

9.Find the disjunctive normal form for a tautology in $p, q.r.$

$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee ($

10. Use truth tables to verify $p \vee (p \wedge q) \equiv p$

p	q	$(p \wedge q)$	$p \vee (p \wedge q)$	p
T	T	T	T	T
T	F	F	T	T
F	T	F	F	F
F	F	F	F	F