Test 2 Math 3336 September 26. 2019

Each problem is worth 20 points! You have the full class period to complete the test. You cannot use books, notes, calculators and phones.

1. Decide whether the following formulas are equivalent. In case where your answer is "not equivalent" you must give an explanation.

(a) $\exists x(Q(x) \wedge P(x))$ and $\exists xQ(x) \wedge \exists xP(x)$ not

(b) $\exists x(Q(x) \vee P(x))$ and $\exists xQ(x) \vee \exists xP(x)$ yes

(c) $\exists x(Q(x) \rightarrow P(x))$ and $\exists Q(x) \rightarrow \exists xP(x)$ not

Solution: In (a) let $Q(x)$ be "x is even" and P(x) be "x is odd". Then the left-hand side is false on ℕ but the right-hand side is true.

For (c), let $D = \{a,b\}$, $Q(a), \neg Q(b), \neg P(a), \neg P(b)$. Then $\exists x(Q(x) \rightarrow P(x))$ is true at b because at b, $Q(b) \rightarrow P(b)$ is true. $\exists Q(x) \rightarrow \exists xP(x)$ is false because $\exists Q(x)$ is true, we have $Q(a)$, but $\exists xP(x)$ is false.

2.Let $P(x), I(x)$, and $V(x)$ be the statements "*x is a professor*", "*x is ignorant*", "x is vain", respectively. Express each of these statements using quantifiers, logical connectives $P(x)$, $I(x)$, and $V(x)$, where the domain consists of all people:

(a) No professors are ignorant or vain. Solution: $\forall x(P(x) \rightarrow \neg I(x) \wedge \neg V(x))$

(b) All ignorant people are vain. Solution: $\forall x(I(x) \rightarrow V(x))$

(c) All professors are vain and ignorant. Solution: $\forall x(P(x) \rightarrow (V(x) \wedge I(x)))$

3. Let $L(x,y)$ be the predicate for "x likes to buy y" and let $S(y)$ stand for "y is on sale". Formalize:

(a) Everybody likes to buy something, but only if it is on sale. Solution: $\forall x \exists y(L(x,y) \rightarrow S(y))$

(b) There is something everybody likes to buy if it is on sale. Solution: $\exists y \forall x(S(y) \rightarrow L(x,y))$

(c)If everything is on sale then everybody likes to buy everything. Solution: $\forall yS(y) \rightarrow \forall x \forall yL(x,y)$

(d) There is somebody who likes to buy everything if it is on sale. Solution: $\exists x \forall yS(y) \rightarrow L(x,y)$

4. Express the negation of these statements so that all negation symbols immediately precede predicates.

(a) $\forall x \exists y \exists zT(x,y,z)$. Solution:$\exists x \forall y \forall z \neg T(x,y,z)$

(b) $\forall x \exists yP(x,y) \vee \forall x \exists yQ(x,y)$ Solution: $\exists x \forall y \neg P(x,y) \wedge \exists x \forall y \neg Q(x,y)$

(c) $\forall x \exists y(P(x,y) \wedge \exists zR(x,y,z))$ Solution: $\exists x \forall y(\neg P(x,y) \vee \forall z \neg R(x,y,z))$

(d)$\forall x \exists y(P(x,y) \rightarrow Q(x,y))$ Solution: $\exists x \forall y(P(x,y) \wedge \neg Q(x,y))$

5. Determine whether the following arguments are valid or invalid. (L you have studied logic, A you can answer this problem)

(a)You can answer this problem only if you have studied the chapter on logic. $A \rightarrow L$
You can answer this problem.A
Therefore you have studied the chapter on logic. L Solution:
$(A \wedge (A \rightarrow L)) \rightarrow L$, tautology; correct

(b)If you have studied the chapter on logic then you can answer this problem.$L \rightarrow A$
You can't answer this problem $\neg A$.

Therefore you have not studied the chapter on logic.$\neg L$ correct reasoning
(c)If you have studied the chapter on logic then you can answer this problem.$L \to A$
You can answer this problem. A
Therefore you have studied the chapter on logic.L incorrect reasoning
(d) If you have studied the chapter on logic then you can answer this problem.$L \to A$
You have studied the chapter on logic L
Therefore you can answer the problem. A correct

6. Justify the following rule of inference by showing that a certain formula is a tautology.

$$\begin{array}{c} p \lor q \\ \underline{\neg p \lor r} \\ q \lor r \end{array}$$

$((p \lor q) \land (\neg p \lor r)) \ \to (q \lor r)$ is a tautology.