November 14, 2019

Test 4, Math 3336

Each problem is worth 20 points. You cannot use books, notes, phones or computers. You have the full class period to complete the test.

1. Use mathematical induction to show that 3 divides $n^3 + 2n$. Solution:
$n = 1 : 3|1^3 + 2 \cdot 1 = 3;$ assume $3|n^3 + 2n$ we have
$(n+1)^3 = n^3 + 3n^2 + 3n + 1, 2 \cdot (n+1) = 2 \cdot n + 2,$
$(n+1)^3 + 2(n+1) = n^3 + 3n^2 + 3n + 1 + 2n + 2 = n^3 + 2n + 3(n^2 + n + 1)$ which is divisible by 3

2. Use mathematical induction to show that $1 + 3 + 5 + \cdots + (2n+1) = (n+1)^2$
Solution:
$n = 1 : 1 + 3 = 4 = (1+1)^2; n+1 : 1 + 3 + 5 + \cdots + (2n+1) + (2n+3) = (n+1)^2 + 2n + 3 =$

3. Prove that the divides relation on the set of positive integers is transitive. Solution::
Assume $a|b$ and $b|c$. Show $a|c$. We have $ak = b, bl = c$ therefore $(ak)l = bl = c; a(kl) = c$

4. List all the ordered pairs in the relation $R = \{a,b)|a$ divides $b\}$ on the set $\{1,2,3,4,6\}$
$R = \{(1,1),(1,2),(1,3),(1,4),(1,6).(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)\}$

5. Consider the following relation on the set
$\{1,2,3,4\} : R = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)$. Find
a) the reflexive closure of $R; R \cup \Delta = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4) \cup \{\{3,3)\}$
b) the symmetric closure of
$R : R \cup R^{-1} = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)\} \cup \{(1,1),(2,1),(1,2),(2,2),(4,3)$
c) the transitive closure of
$R.R^* = \{(1,1),(1,2),(1,3),(2,2),(2,1),(3,4),(3,1),(4,4),(4,1)\}$

6. Find the partition of the smallest equivalence relation that contains R of the previous exercise. $\pi = \{1,2,4,1\}$

7. Let R be the relation on the set of real numbers such that aRb if and only if $a - b$ is an integer. Show that R is an equivalence relation. What is the class of 1? solution: aRa because $a - a = 0; a - b = n$ then $b - a = -n;$ if $a - b = n$ and $-c - b = m$ then
$-c - a = (c - b) + (b - a) = m - n$

8. Which of these pairs of elements are comparable in the poset $(\mathbb{N}, |)$?
a)$5, 15$ b)$6, 9$ c)$8, 16$ d)$7, 7$ we have $5|15, 8|18$ and $7|7$

9. Give the definition of a lattice. Determine whether these posets are lattices.
In a lattice any two elements a and b have a least upper bound and a largest lower bound.

a) $(\{1,3,6,9,12\},|)$ NO
b) $\{1,5,25,125],|)$ Yes
c) $(\mathbb{Z},\leq)$ Yes
d) $(P(S),\subseteq)$ where $P(S)$ is the powerset of a set S yes.

10. Draw the order diagram for divisibility of the set $\{1,2,3,4,5,6\}$.