Modus Ponens, Rules of Inference

Many logical arguments are based on a rule which is known as *modus ponens* or rule of detachment. Assume that p is true and that $p \to q$ is true. Then you can conclude q. Formally:

$$\begin{array}{c} p \\ \underline{p \to q} \\ q \end{array}$$

here are some examples involving this rule:

p : It is September.

q : Houston will get a cool-front then

$p \to q$ In September, Houston gets a cool -front.

Thus Houston will get a cool-front this month.

This is correct reasoning.

Let:

p : You have studied logic.

$p \to q$: if you have studied logic then you can answer this problem.

Thus you can answer this problem. Again, this is correct reasoning.

While: If you have studied logic then you can answer this problem. You can answer this problem Thus you have studied logic. This is incorrect reasoning.

$p \to q$ and q, yields p is incorrect reasoning. You may already know the solution without having studied logic.

Correct reasoning is to conclude $\neg p$ from from $\neg q$ and $p \to q$. Which is

$$\begin{array}{c} \neg q \\ \underline{p \to q} \\ \neg p \end{array}$$

This version of modus ponens is called *modus tollens*:

You cannot answer this problem. If you had studied logic then you could have answered the problem. So you did not study logic. This is correct reasoning according to modus tollens.

The following are rules involving quantifiers:

$$\frac{\forall x P(x)}{P(c)}$$

Here c is an arbitrary element of the domain where $\forall x P(x)$ is true. This is called *Universal instantiation*. While

$$\frac{P(c)}{\forall x P(x)}$$

is called *Universal generalization*

IF one can show that $P(c)$ is true for any c of the domain then $\forall x P(x)$ must be true.

Similarly,

$$\frac{\exists x P(x)}{P(c)}$$

Here c is a specific c for which $P(c)$ is true. This is called *Existential instantiation.* And

$$\frac{P(c)}{\exists x P(c)}$$

is called *Existential generalization.* Here c is a particular element of the domain for which $P(c)$ is true.

While quite obvious, these rules make up a complete system