Test 3.Math 5330 March 25

This is a take home test. I assume that you adhere to the Academic Honors code and work alone. You can use the text book but not your computer for answers. Mail your answers back to

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1. Prove that a subgroup of a cyclic group is cyclic.

2. What are the subgrups of $\mathbb{Z}$ with addition?

3. Prove that if G=<x> then G=<x^{-1}>

4. Prove that if G=<x> and and G is infinite then x and x^{-1} are the only generators of G.

5. Show that it is impossible for a group G to be the union of two proper subgroups.

6. Let G be a group and let $g \in G$.Show that $Z(g)=\{x|xg=gx\}$is a subgroup of G.

7. Let G be a group and let $a \in G$ Define a function $f:G \to G$ by $f(x)=axa^{-1}$..Prove that f is bijective (one-one and onto)

8. Let X be a set .and let $Y \subseteq X$. Show that the subset of all permutations of X consisting of all f such that $f(y)=y$ for all $y \in Y$ forms a subgoup.

9. Is the group Q of rational numbers cyclic?

10. Show that the intersection of two subgroups of a group G is itself a subgroup of G..