

## Homework 6      Math 6302

1. Let  $\mathbf{A} = (A, (f_t)_{t \in T})$  be an algebraic system and let  $M$  be a subset of  $A$ . Let  $P = P(x)$  be a property concerning elements of  $\mathbf{A}$ . Assume that
  - (a) Every element of  $M$  has property  $P$ .
  - (b) If  $f_t$  is an  $n$ -ary operation and if  $a_0, \dots, a_{n-1}$  have property  $P$ , then  $a = f_t(a_0, \dots, a_{n-1})$  has property  $P$ .

Claim: Every element of the closure  $\mathcal{C}(M)$  has property  $P$ . This is called “Proof by Structural Induction”.

2. Algebraic expressions are defined inductively as:
  - (a) Every natural number  $n \in \mathbb{N}$  is an algebraic expression
  - (b) Assume that  $\alpha$  and  $\beta$  are algebraic expressions. Then
    - i.  $(\alpha + \beta)$  is an algebraic expression.
    - ii.  $(\alpha \cdot \beta)$  is an algebraic expression.

All algebraic expressions are obtained by finitely many applications of (a) and (b). Prove by structural induction that every algebraic expression has the same number of left parenthesis as right parenthesis. (You may have to do some formalization.)

3. Do the following problems in Hungerford:  
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