4. A cylindrical glass of radius $r$ and height $L$ is filled with water and then tilted until the water remaining in the glass exactly covers its base.
   (a) Determine a way to "slice" the water into parallel rectangular cross-sections and then set up a definite integral for the volume of the water in the glass.
   (b) Determine a way to "slice" the water into parallel cross-sections that are trapezoids and then set up a definite integral for the volume of the water.
   (c) Find the volume of water in the glass by evaluating one of the integrals in part (a) or part (b).
   (d) Find the volume of the water in the glass from purely geometric considerations.
   (e) Suppose the glass is tilted until the water exactly covers half the base. In what direction can you "slice" the water into triangular cross-sections? Rectangular cross-sections? Cross-sections that are segments of circles? Find the volume of water in the glass.

22. The velocity $v$ of blood that flows in a blood vessel with radius $R$ and length $l$ at a distance $r$ from the central axis is

$$v(r) = \frac{P}{4\eta l} \left( R^2 - r^2 \right)$$

where $P$ is the pressure difference between the ends of the vessel and $\eta$ is the viscosity of the blood (see Example 7 in Section 3.7). Find the average velocity (with respect to $r$) over the interval $0 \leq r \leq R$. Compare the average velocity with the maximum velocity. (Find the ratio of the average and maximum velocities).

79. Use the substitution $u = 1/x$ to show that

$$\int_0^\infty \frac{\ln x}{1 + x^2} \, dx = 0$$
5. (a) Show that the volume of a segment of height $h$ of a sphere of radius $r$ is

$$V = \frac{1}{3}\pi h^2(3r - h)$$

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9. The figure shows a curve $C$ with the property that, for every point $P$ on the middle curve $y = 2x^2$, the areas $A$ and $B$ are equal. Find an equation for $C$.

10. A paper drinking cup filled with water has the shape of a cone with height $h$ and semivertical angle $\theta$ (see the figure). A ball is placed carefully in the cup, thereby displacing some of the water and making it overflow. What is the radius of the ball that causes the greatest volume of water to spill out of the cup?
46. Suppose you make napkin rings by drilling holes with different diameters through two wooden balls (which also have different diameters). You discover that both napkin rings have the same height $h$, as shown in the figure.
(a) Guess which ring has more wood in it.
(b) Check your guess: Use cylindrical shells to compute the volume of a napkin ring created by drilling a hole with radius $r$ through the center of a sphere of radius $R$ and express the answer in terms of $h$.

78. Find the value of the constant $C$ for which the integral

$$\int_{0}^{\infty} \left( \frac{x}{x^2 + 1} - \frac{C}{3x + 1} \right) dx$$

converges. Evaluate the integral for this value of $C$.

(a) Prove that if $f$ is a continuous function, then

$$\int_{0}^{a} f(x) \, dx = \int_{0}^{a} f(a - x) \, dx$$

(b) Use part (a) to show that

$$\int_{0}^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} \, dx = \frac{\pi}{4}$$

for all positive numbers $n$. 