Math 1330 - Section 2.1
Linear and Quadratic Functions

Recall: Equation of a line:
General form: \( ax + by = c \), (slope is: \( m = -\frac{a}{b} \))
Slope-intercept form: \( y = mx + b \) (\( m \) is the slope and \( b \) is the \( y \)-intercept)
Point-slope form: \( y - y_1 = m(x - x_1) \)

If two points on the line are given, then the slope is: \( m = \frac{y_2 - y_1}{x_2 - x_1} \)

Vertical lines are of the form: \( x = c \).
Horizontal lines are of the form: \( y = c \).

Two lines are parallel if they have the same slope.
Two lines are perpendicular if their slopes are negative reciprocals of each other.

A linear function is a function of the form \( f(x) = mx + b \), where \( m \) is the slope and \( b \) is the \( y \)-intercept.

Example 1: Write an equation of the linear function for which \( f(2) = 5 \) and \( f(-1) = -2 \).

Example 2: Write an equation of the linear function which contains the point \((2, -5)\) and whose inverse contains the point \((-1, 6)\).

Example 3: Write an equation of the linear function which is parallel to the line \( 2x - 5y = 10 \) and which passes through the point \((-1, -4)\).

A quadratic function is a function of the form \( f(x) = ax^2 + bx + c \), \( a \neq 0 \).
The graph of a quadratic function is called a parabola. You should be able to identify the following features of the graph of a quadratic function:

- direction the graph opens (upward or downward)
- whether the function has a maximum or a minimum
- \( y \) intercept \( (f(0).) \)
- coordinates of the vertex
- equation of the axis of symmetry
- maximum or minimum value

If \( a > 0 \), the parabola will open upward. In this case, the function has a minimum value. If \( a < 0 \), the parabola will open downward. In this case, the function has a maximum value.

**The standard form of a quadratic function:**

\[
f(x) = a(x - h)^2 + k\quad \text{is in the standard form.}
\]

The vertex is \((h, k)\) and the axis of symmetry is the line \( x = h \).

The maximum or minimum value of the function is the number \( k \) (the \( y \)-coordinate of the vertex).

**Example 4:** Given the function \( f(x) = -2x^2 - 10x + 6 \).
Find the standard form:

Find the vertex:

Find the axis of symmetry:

State the max/min value:

NOTE: If you are not asked to write the function in standard form, you can find the vertex using a different method. The coordinates of the vertex of the graph of the function

\[ f(x) = ax^2 + bx + c, \ a \neq 0 \]

is the ordered pair \( \left( \frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right) \).

If you are given the vertex of the graph of a function and another point, you can find the quadratic function equation.

Example 5: Write the equation of the quadratic function which passes through the point \((0, 7)\) and whose vertex is \((-2, 10)\).

Example 6: Find the quadratic function that satisfies:
The axis of symmetry is: $x = -4$

The y-intercept is: $(0,80)$

There is only one $x$-intercept.

**Example 7:** A rocket is fired directly upwards with a velocity of 80 ft/sec. The equation for its height, $H$, as a function of time, $t$, is given by the function $H(t) = -16t^2 + 80t$.

a. Find the time at which the rocket reaches its maximum height.

b. Find the maximum height of the rocket.