Math 1330 – Section 8.2
Ellipses

Definition: An ellipse is the set of all points, the sum of whose distances from two fixed points is constant. Each fixed point is called a focus (plural = foci).

Basic ellipses (centered at origin):

Basic “vertical” ellipse:
Equation: \( \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \ a > b \)
Foci: \((0, \pm c)\), where \(c^2 = a^2 - b^2\)
Vertices: \((0, \pm a)\)
Eccentricity: \(e = \frac{c}{a}\)

Basic “horizontal” ellipse:
Equation: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \ a > b \)
Foci: \((\pm c, 0)\), where \(c^2 = a^2 - b^2\)
Vertices: \((\pm a, 0)\)
Eccentricity: \(e = \frac{c}{a}\)

The eccentricity provides a measure on how much the ellipse deviates from being a circle. The eccentricity \(e\) is a number between 0 and 1.
- small \(e\): graph resembles a circle (foci close together)
- large \(e\): flatter, more elongated (foci far apart)
- if the foci are the same, it’s a circle!
Graphing ellipses:

To graph an ellipse with center at the origin:

- Rearrange into the form \( \frac{x^2}{\text{number}} + \frac{y^2}{\text{number}} = 1 \).

- Decide if it’s a “horizontal” or “vertical” ellipse.
  - if the bigger number is under \( x^2 \), it’s horizontal (longer in \( x \)-direction).
  - if the bigger number is under \( y^2 \), it’s vertical (longer in \( y \)-direction).

- Use the square root of the number under \( x^2 \) to determine how far to measure in \( x \)-direction.

- Use the square root of the number under \( y^2 \) to determine how far to measure in \( y \)-direction.

- Draw the ellipse with these measurements. Be sure it is smooth with no sharp corners.

- \( c^2 = a^2 - b^2 \) where \( a^2 \) and \( b^2 \) are the denominators. So \( c = \sqrt{\text{big denom} - \text{small denom}} \)

- The foci are located \( c \) units from the center on the long axis.

To graph an ellipse with center not at the origin:

- Rearrange (complete the square if necessary) to look like \( \frac{(x-h)^2}{\text{number}} + \frac{(y-k)^2}{\text{number}} = 1 \).

- Start at the center \((h,k)\) and then graph it as before.

When graphing, you will need to find the orientation, center, values for \( a \), \( b \) and \( c \), vertices, foci, lengths of the major and minor axes and eccentricity.

**Example 1:** Find all relevant information and graph \( \frac{x^2}{16} + \frac{y^2}{9} = 1 \).
Example 2: Find all relevant information and graph \( \frac{(x-1)^2}{9} + \frac{(y+2)^2}{25} = 1 \).
Example 3: Write the equation in standard form. Find all relevant information and graph:
\[4x^2 - 8x + 9y^2 - 54y = -49.\]

Example 4: Find the equation for the ellipse satisfying the given conditions.
Foci \((\pm 3, 0)\), vertices \((\pm 5, 0)\)

Example 5: Write an equation of the ellipse with vertices \((5, 9)\) and \((5, 1)\) if one of the foci is \((5, 7)\).