1 Thursday, May 31

Lecture on 1.5: Matrix Multiplication, Elementary matrices An elementary matrix \(E\) is a (necessarily square) matrix which is obtained from an identity matrix \(I_m\) by a single ERO. Denote \(e\) be such an ERO, then \(E = e(I_m)\) (which is a \(m \times m\) matrix). Denote by \(e\) the ERO. It is important to view that “premultiplication as row manipulation, i.e. performing an ERO for an \(m \times n\) matrix \(A\) is the same multiplying the corresponding elementary matrix \(E\) to the left of the matrix \(A\), i.e. \(e(A) = EA\). (Theorem 9 on Page 20). A consequence of this is the fact that an \(m \times n\) matrix \(B\) is row equivalent to \(A\) iff \(B = PA\) where \(P\) is a product of finitely many \(m \times m\) elementary matrices. In particular, for any \(m \times n\) matrix \(A\), \(PA = R\) where \(P\) is a product of finitely many \(m \times m\) elementary matrices, and \(R\) is the row reduced echelon matrix. This greatly simplified the matrix \(A\).

Lecture on 1.6: Invertible Matrices We discuss properties of invertible matrices. We also talked about the invertibility of a matrix \(A\) is related to the solvability of the system of equations \(AX = Y\) (see Theorem 13 on P. 23).