1. Prove that if $n$ is an odd prime, then $n > 1$.

2. Show that if $n$ is even, then $n > 1$.

3. Find the smallest positive integer $n$ with the given value of $d$.
   - $d = (u)p$ (p)
   - $d = (u)p$ (q)
   - $d = (u)p$ (a)

4. Given a list of the positive divisors of $n$.
   - List the prime factorization from part (a) to compute $p(r)$ (u)
   - List the prime factorization from part (a) to compute $p(r)$ (a)

5. For each of $u = 24$, $n = 105$, and $n = 544$.
Chapter I. Divisibility and Factorization

20. Let n be a positive integer.

21. Prove or disprove: \( a + b = a + b \).

22. Prove or disprove: \( \frac{a}{b} = \frac{a}{b} \).

23. Prove or disprove: \( a - b = a - b \).

24. Prove or disprove: \( a \times b = a \times b \).

25. Prove or disprove: \( a \div b = a \div b \).

26. Prove or disprove: \( a \mod b = a \mod b \).

27. Prove or disprove: \( a \equiv b \mod m \).

28. Prove or disprove: \( \gcd(a, b) = \gcd(a, b) \).

29. Prove or disprove: \( \lcm(a, b) = \lcm(a, b) \).

30. Prove or disprove: \( a \mid b \) if and only if \( \gcd(a, b) = a \).

31. Prove or disprove: \( a \mid b \) if and only if \( \lcm(a, b) = b \).

32. Prove or disprove: \( a \mid b \) if and only if \( \gcd(a, b) = 1 \).

33. Prove or disprove: \( a \mid b \) if and only if \( \lcm(a, b) = ab \).

34. Prove or disprove: \( a \mid b \) if and only if \( \gcd(a, b) = ab \).

35. Prove or disprove: \( a \mid b \) if and only if \( \lcm(a, b) = a \).

36. Prove or disprove: \( a \mid b \) if and only if \( \gcd(a, b) = b \).

37. Prove or disprove: \( a \mid b \) if and only if \( \lcm(a, b) = b \).

38. Prove or disprove: \( a \mid b \) if and only if \( \gcd(a, b) = a \).

39. Prove or disprove: \( a \mid b \) if and only if \( \lcm(a, b) = ab \).

40. Prove or disprove: \( a \mid b \) if and only if \( \gcd(a, b) = 1 \).

41. Prove or disprove: \( a \mid b \) if and only if \( \lcm(a, b) = ab \).

42. Prove or disprove: \( a \mid b \) if and only if \( \gcd(a, b) = b \).

43. Prove or disprove: \( a \mid b \) if and only if \( \lcm(a, b) = a \).

44. Prove or disprove: \( a \mid b \) if and only if \( \gcd(a, b) = a \).

45. Prove or disprove: \( a \mid b \) if and only if \( \lcm(a, b) = ab \).

46. Prove or disprove: \( a \mid b \) if and only if \( \gcd(a, b) = 1 \).

47. Prove or disprove: \( a \mid b \) if and only if \( \lcm(a, b) = b \).

48. Prove or disprove: \( a \mid b \) if and only if \( \gcd(a, b) = a \).

49. Prove or disprove: \( a \mid b \) if and only if \( \lcm(a, b) = ab \).

50. Prove or disprove: \( a \mid b \) if and only if \( \gcd(a, b) = b \).

51. Prove or disprove: \( a \mid b \) if and only if \( \lcm(a, b) = a \).
33. Two integers \( m \) and \( n \) are called an amicable pair if \( (m, n) \) is perfect.

Show that if \( 2^k - 1 \) is prime, then \( n = 2^{k-1}(2^k - 1) \) is perfect.

(34) An integer \( n \) is perfect if \( \sigma(n) = 2n \).

\[
(0 < w) \sum_{\omega \mid w} = (w)^\omega \quad (q)
\]

\[
\prod_{\omega \mid w} = (w)^\omega \quad (a)
\]

Find a general formula for:

33. Suppose that \( u \) is multiplicative.

Show that \( (u) \sigma \) is multiplicative.

(32) A function \( f \) defined on the positive integers is said to be multiplicative if

\[
\left( \frac{1 - \frac{r}{d}}{1 - \frac{r}{1+d}} \right) \left( \frac{1 - \frac{s}{d}}{1 - \frac{s}{1+d}} \right) = (u) \sigma
\]

(31) For \( u, w \) with \( \gcd(u, w) = 1 \), show that

\[
(u, w) \sigma \text{ and } (u, w) \tau \text{ are equal to zero. (q)}
\]

It is possible that some of the \( a \)'s and \( b \)'s are equal to zero.

(30) Suppose that \( u = \prod p_i^{a_i} \), \( m = \prod q_j^{b_j} \), \( (u, w) \sigma \text{ and } (u, w) \tau \text{ are } a \text{ and } b \text{ integers, respectively. (q)}

(29) Let \( u \) be a positive integer.

4. \( 2^k - 1 \) is prime.

Show that \( 2^k - 1 \) is prime.

(28) Show that \( 6, 26, \text{ and } 496 \) are all perfect.

\[
(0 < w) \sum_{\omega \mid w} = (w)^\omega \quad (q)
\]

\[
\prod_{\omega \mid w} = (w)^\omega \quad (a)
\]
Deduction of the solution to the given problem:

4. By inspection, find a few integer solutions (if there are any) to the equation 6x + 2y = 4.

2. By inspection, find a few integer solutions (if there are any) to the equation 6x + 2y = 4.

Hint: Determine the gcd of 6 and 2, and consider the solutions of the form ax + by = gcd(6, 2).

The equation depends on the values of a, b, and c. The integer solutions to the equation depend on the integer values of x and y.

1. Find the quotient q and the remainder r from the division algorithm.

2. For each pair a and b, and remainder (computed by you) in the form of ax + by = gcd(6, 2).

3. Determine the gcd(6, 2) and the remainder r from the division algorithm.

4. By inspection, find a few integer solutions (if there are any) to the equation 6x + 2y = 4.

5. A sequence of integers a, b, and c is called an algebraic cycle of length k if

\[ a = a_k = b - (a_{k-1}) \]

Show that the integers \(a_k\) form an algebraic cycle.

Prelab

Propositional Equations

2. The Euclidean Algorithm and Linear