1 (a) \{4\}.
(b) \{0, 1, 2, 3, 4, 5, 7\}.
(c) \{0, 1, 3, 5, 7\}.
(d) \{\emptyset\}.

2 (a) If \(x\) belongs to \(A \cap (B \setminus C)\) then \(x\) belongs to \(A\) and to \(B\) but not to \(C\). It follows that \(x\) belongs to \(A \cap B\) and \(A \cap C\) is empty. It follows that \(x \in (A \cap B) \setminus (A \cap C)\).

Conversely, if \(x \in (A \cap B) \setminus (A \cap C)\) then it means that \(x \in A \cap B\) but not in \(A \cap C\). It means that \(x \in A\) and \(x \in B\) but not in \(C\). So, \(x \in A\) and \(x \in (B \setminus C)\), and it follows that \(x \in A \cap (B \setminus C)\).

(b) similar to part a.
(c) Not true. If \(C\) contains a point in common with \(B\) then such point will be in the set on the left, but not on the right.

3 \(A^c \cap B^c\) equals \((A \cup B)^c\). So, in general, \(A^c\) and \(B^c\) are not disjoint. \(A \cap C\) and \(B \cap C\) are disjoint since the first is the subset of \(A\) and the second is the subset of \(B\).

4 \[ p(A \cup B) \geq p(A) = 1. \]

For any events \(p(A \cup B) \leq 1\). It follows that \(p(A \cup B) = 1\). Now, from the formula

\[ p(A \cup B) = p(A) + p(B) - p(A \cap B), \]

we get \(p(A \cap B) = 1\). Since

\[ p(A \setminus B) = p(A) - p(A \cap B) = 1 - 1 = 0. \]
Let $A$ denote the event that the midtown temperature in Los Angeles is 70F, and let $B$ denote the event that the midtown temperature in New York is 70F. Also, let $C$ denote the event that the maximum of midtown temperatures in New York and Los Angeles is 70F. If 

\[ p(A) = .3, \quad p(B) = .4, \quad p(C) = .2, \]

find the probability that the minimum of the two midtown temperatures is 70F.

Solution. Consider $A \cap B$ and $A \cup B : A \cap B$ means that both temperatures are 70. It true that $A \cap B$ is contained in the set $D$ = \{ minimum of temperatures is 70\}, but they might not be same. However, $A \cap B$ is the same as $C \cap D$; if min and max is 70 then both are 70.

Also, $A \cup B$ is the same as $C \cup D$; the first set means that at least one temp. is 70 and it is the same as to say that either max is 70 or min is 70 – which is the second set.

So we write:

\[ p(A \cup B) = p(A) + p(B) - p(A \cap B), \]

or

\[ p(C \cup D) = p(A) + p(B) - p(C \cap D). \]

But also,

\[ p(C \cup D) = p(C) + p(D) - p(C \cap D). \]

Subtract this equation from the previous and find $p(D)$.

\[ p(D) = .5. \]