Math 1310

Section 2.8: Absolute Value

In this lesson, you’ll learn to solve absolute value equations and inequalities.

Definition: The absolute value of \( x \), denoted \(|x|\), is the distance \( x \) is from 0.

Solving Absolute Value Equations
If \( C \) is positive, then \(|x| = C \) if and only if \( x = \pm C \).

Special Cases for \(|x| = C\):
Case 1: If \( C \) is negative then the equation \(|x| = C \) has no solution since absolute value cannot be negative.
Case 2: The solution of the equation \(|x| = 0 \) is \( x = 0 \).

Example 1: Solve.

a. \(|2x - 3| = 7\)

b. \(|6 - 2x| + 6 = 14\)

c. \(2|-3(2x - 8)| + 4 = 30\)

d. \(-4\left|\frac{1}{2}x + 1\right| + 3 = -11\)
Next, we’ll look at inequalities. The approach to these problems will depend on whether the problem is a “less than” problem or a “greater than” problem. If $C$ is zero, then $x = 0$.

**Solving Absolute Value Inequalities**

If $C$ is positive, then

a. $|x| < C$ if and only if $-C < x < C$.

b. $|x| ≤ C$ if and only if $-C ≤ x ≤ C$.

c. $|x| > C$ if and only if $x > C$ or $x < -C$.

d. $|x| ≥ C$ if and only if $x ≥ C$ or $x ≤ -C$

**Example 2:** Solve the inequality. Graph the solution on the real number line. Write the solution using interval notation:

a. $|x + 3| ≤ 8$

b. $|4 - 2x| < 12$

c. $3|2x - 6| ≤ 6$

d. $|−3x + 1| < 4$
e. $2|x - 4| + 1 > 7$

f. $\frac{2}{3}|x - 4| \leq -\frac{4}{3}$

**Special Cases:**

**Case 1:**

If $C$ is negative, then:

a) The inequalities $|x| < C$ and $|x| \leq C$ have no solution.

b) Every real number satisfies the inequalities $|x| > C$ and $|x| \geq C$

**Case 2:**

a) The inequality $|x| < 0$ has no solution.

b) The solution of the inequality $|x| \leq 0$ is $x = 0$.

c) Every real number satisfies the inequality $|x| \geq C$