Online Math 1314 Final Exam Review

1. The following table of values gives a company's annual profits in millions of dollars. Rescale the data so that the year 2003 corresponds to $x = 0$.

$x=$	0	1	2	3	4	5
Year	2003	2004	2005	2006	2007	2008
Profits (in millions of dollars)	31.3	32.7	31.8	33.7	35.9	36.1

$>$ list1

Begin by creating a list.

a. Find the cubic regression model for the data.

Command: fitpoly[list1, 3]

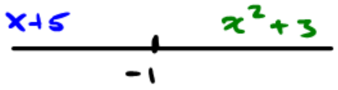
Answer: $f(x) = -0.0556x^3 + 0.531x^2 - 0.3183x + 31.5952$

b. Use the cubic regression model to predict the company's profits in 2010. ← $t = 7$

Command: $f(7)$

Answer: 36.3238 million

2. Suppose $f(x) = \begin{cases} x+5, & x \le -1 \\ x^2+3, & x > -1 \end{cases}$



Determine, if they exist,

A. $\lim_{x \to -1^-} f(x)$

Left side of -1

$-1+5 = 4$

B. $\lim_{x \to -1^+} f(x)$

Right side of -1

$(-1)^2+3 = 4$

C. $\lim_{x \to -1} f(x)$

Left = Right

$4 = 4$

3. $\lim_{x \to 1} \frac{\sqrt{4x}}{x-5} = \frac{\sqrt{4(1)}}{1-5} = \frac{\sqrt{4}}{-4} = -\frac{2}{4} = -\frac{1}{2}$

4. $\lim_{x \to 4} \frac{x+3}{2x-8} = \frac{4+3}{2(4)-8} = \frac{7}{0} =$ undefined

5. $\lim_{x \to -2} \frac{x^2+5x+6}{x+2} = \text{“}\frac{0}{0}\text{”}$

Command: limit[f, -2]

Answer: 1

6. $\lim_{x\to\infty} \frac{10x^2 - x}{3 - 4x^2}$

Same Degrees

$\frac{10}{-4} = \boxed{\frac{-5}{2}}$

7. $\lim_{x\to -\infty} \frac{x^3 + 5x^2 - 7x - 1}{2 + x^2 - 7x^4}$

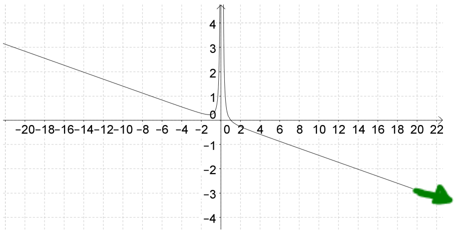
Bottom is Bigger Degree

$= \boxed{0}$

8. $\lim_{x\to\infty} \frac{x^3 - 1}{x - 7x^2}$

Top is Bigger

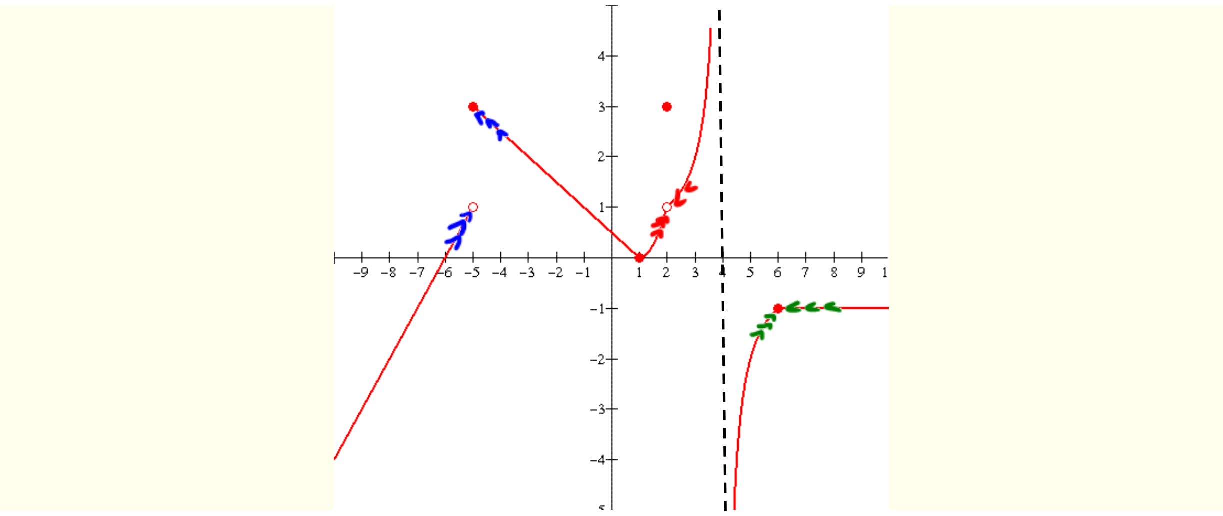
Enter the function into GGB.



$\boxed{\begin{matrix}\text{DNE} \\ \text{or} \\ -\infty\end{matrix}}$

one or the other listed not both

9. The graph of $f(x)$ is shown below. Which of the following statements is true?



I. $\lim_{x\to 2} f(x)$ exists and is equal to 3. $\lim_{x\to 2} f(x) = 1$ So False

II. $\lim_{x\to -5} f(x)$ exists and is equal to 3. Two Different values False

III. $\lim_{x\to 6} f(x)$ does not exist. Approaching same # False

IV. $\lim_{x\to 2} f(x)$ does not exist; there is a hole where $x = 2$. False lim does exist

V. $\lim_{x\to 4} f(x)$ does not exists; there is unbounded behavior as x approaches 4.

True

10. Find the first and second derivative: $f(x)=5x^4-3x^3+8x^2+7x-1$

$f'(x)= 20x^3 - 9x^2 + 16x + 7$

$f''(x)= 60x^2 - 14x + 16$

11. Let $f(x)=3x^4-5x^2+7x-1$. Find the equation of the tangent line at x = 1.

Command:	Answer:
tangent[1,f]	$y = 9x - 5$

12. A ball is thrown upwards from the roof of a building at time t = 0. The height of the ball in feet is given by $h(t) = -16\,t^2 + 148\,t + 78$, where t is measured in seconds. Find the velocity of the ball after 3 seconds.

Derivative of h(x)

$h'(3) =$ 52ft/sec

13. Suppose a manufacturer has monthly fixed costs of \$250,000 and production costs of \$24 for each item produced. The item sells for \$38. Assume all functions are linear. State the:

a. cost function.

$C(x)=cx+F$ $= 24x + 250{,}000$

b. revenue function.

$R(x)=sx$ $= 38x$

c. profit function.

$P(x)=R(x)-C(x)$ $= 38x - [24x+250{,}000] = 14x - 250{,}000$

d. Find the break-even point. (Break Even Quantity, Break-Even Revenue)

$R(x) = C(x)$

$38x = 24x + 250000$

$14x = 250000$

$x = 17857$

$R(17857) = 38(17857)$

$= 678566$

or Type functions into GBB

intersect[R,C]

$\boxed{(17857\ ,\ 678566)}$

14. A clothing company manufactures a certain variety of ski jacket. The total cost of producing x ski jackets and the total revenue of selling x ski jackets are given by the following equations

$$C(x) = 27{,}000 + 22x - 0.25x^2$$
$$R(x) = 500x - 0.1x^2$$
$$0 \le x \le 1{,}000$$

a. Find the profit function.
Recall: P(x) = R(x) – C(x)

$= 500x - 0.1x^2 - [27000 + 22x - 0.25x^2]$

$= 0.15x^2 + 478x - 27000$

b. Find the marginal profit function.

$P`(x) = 0.30x + 478$

$x = 200$

c. Use the marginal profit to approximate the actual profit realized on the sale of the 201st ski jacket.

$P`(200) = 538$

15. At the beginning of an experiment, a researcher has 569 grams of a substance. If the halflife of the substance is 12 days, how many grams of the substance are left after 21 days?
Begin by making a list.

a. State the two points given in the problem.

$(0, 569)$ $\left(12, \frac{569}{2}\right)$

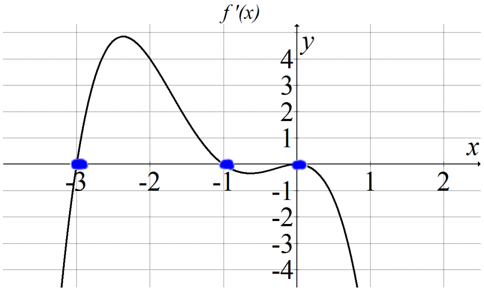
Enter the two points in the spreadsheet and make a list.

Command:	Answer:
fitexp [list1]	$= 569e^{-0.0578x}$

Command:	Answer:
$f(21)$	$= 169.1647$ grams

16. Let $f(x) = -0.2x^5 - x^4 - x^3 - 5$. *Enter the function in GG.*

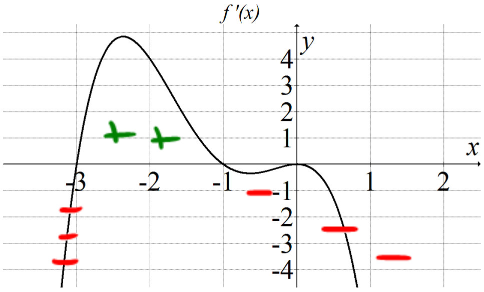
a. Find any critical numbers of *f*. *Find the function's first derivative.*

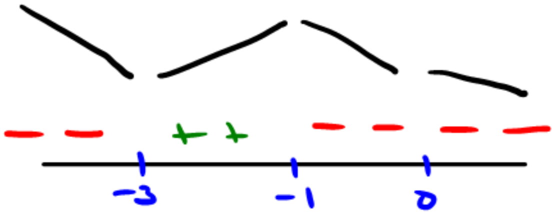


← f '(x)

Command:	Answer:
root [f'(x)]	x = -3 , -1 , 0

b. Interval(s) on which *f* is increasing; interval(s) on which *f* is decreasing.





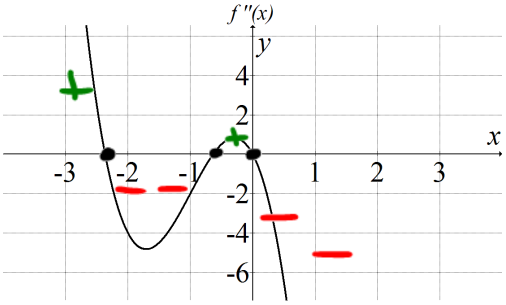
Increasing: (-3, -1)

Decreasing: $(-\infty, -3) \cup (-1, \infty)$

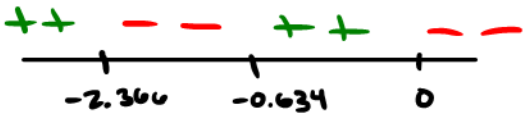
c. Coordinates of any relative extrema.

Command:	Answer:
extremum [f]	(-3, 10.4) R. Min (-1, -4.8) R. Max

d. Interval(s) on which f is concave upward; interval(s) on which f is concave downward.
Find the function's second derivative.



$f''(x)$



Command:
root [f''(x)]

Answer:
$x =$ -2.366
-0.634
0

Concave up: $(-\infty, -2.366) \cup (-0.634, 0)$

Concave down: $(-2.366, 0.634) \cup (0, \infty)$

e. Coordinates of any inflection points.

Command:
inflection point [f]

Answer:
(-2.366, -8.2637)
(-0.634, -4.8863)
(0, -5)

17. Let $f(x) = \dfrac{45}{1 + 2e^{-7x}}$. Find Riemann sums with midpoints and 6 subdivisions to approximate the area between the function and the x-axis on the interval [1, 9]. Then approximate the area between the curve and the x-axis using upper sums with 50 rectangles on the interval [-2, 2],

Recall: "Position of rectangle start": 0 corresponds to left endpoints, 0.5 corresponds to midpoints and 1 corresponds to right endpoints.

Enter the function into GGB.

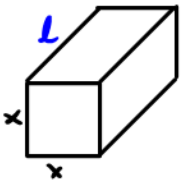
Command:
rectangle Sum [f, 1, 9, 6, 0.5]

Answer:
359.999

Command:
Upper sums [f, 1, 9, 50]

Answer:
359.9936

18. Postal regulations state that the girth plus length of a package must be no more than 104 inches if it is to be mailed through the US Postal Service. You are assigned to design a package with a square base that will contain the maximum volume that can be shipped under these requirements. What should be the dimensions of the package? (Note: girth of a package is the perimeter of its base.)



$4x + l = 104$

$l = 104 - 4x$

$V = l \cdot w \cdot h$

$V = l \cdot x^2$

$V(x) = (104-4x) \cdot x^2$

root [V'(x)]

$x = 0, \ 17.3\bar{3}$

Not Going to work

$V''(17.\bar{3}) < 0$

R. Max

$l = 104 - 4(17.\bar{3})$

$= 34.6\bar{6}$

$\boxed{17.\bar{3} \times 17.\bar{3} \times 34.\bar{6}}$

19. Evaluate the following. $\int_{1.3}^{1.5} \frac{6.95 x^2}{\sqrt{3.65x - 1.95}} \, dx$

Enter the function into GGB.

Command: integral [f, 1.3, 1.5]

Answer: 1.5335

20. A company estimates that the value of its new production equipment depreciates at the rate of $\frac{dv}{dt} = 10000\,(t-9)$ $0 \le t \le 9$, where v gives the value of the equipment after t years. Find the total decline in value of the equipment over the first 5 years.

Enter the function into GGB.

a. Setup the integral needed to answer the question.

$$\int_0^5 10000(x-9)\,dx$$

b. Find the total decline in value of the equipment over the first 5 years.

Command: integral [f, 0, 5]

Answer: -325,000

Declined $325000

21. The temperature in Minneapolis over a 12 hour period can be modeled by the function $C(t) = -0.06t^3 + 0.2t^2 + 3.7t + 5.3$ where t is measured in hours with $t = 0$ corresponding to the temperature at 12 noon. Find the average temperature during the period from noon until 7 p.m.

Recall: $\frac{1}{b-a}\int_a^b f(x)dx$

Enter the function into GGB.
a. Set-up the integral needed to answer the question.

$$\frac{1}{7-0}\int_0^7 (-0.06x^3 + 0.2x^2 + 3.7x + 5.3)\,dx$$

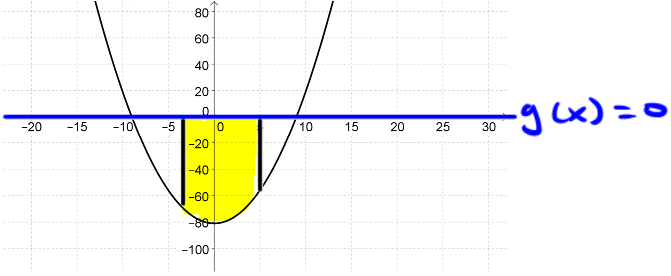
b. Find the average temperature during the period from noon until 7 p.m.
Command: 1/7 * integral[f, 0, 7]

Answer: 16.37

22. Find the area bounded by the graph of $f(x) = x^2 - 81$, the x-axis and the lines $x = -3$ and $x = 5$.

Recall: The general "formula" for computing the area between two curves is
$\int_a^b (\text{top function - bottom function})dx$. Top Bottom
The command is: IntegralBetween[<Function>, <Function>,<Start x-Value>, <End x-Value>]

Enter the function into GGB.



a. Set-up the integral needed to calculate the desired area.

$$\int_{-3}^{5} (g - f)\,dx = \int_{-3}^{5} (-x^2 + 81)\,dx$$

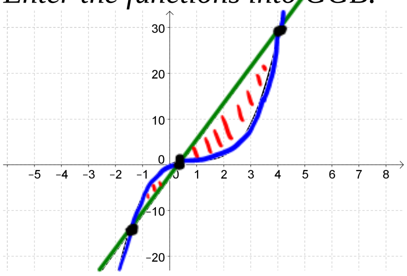
b. Calculate the area.
Command:

Answer:

integralbetween[g, f, -3, 5] = 597.333

23. Find the area of the region(s) that is/are completely enclosed by the graphs of $f(x) = (x-1)^3 + 1$ and $g(x) = 8x - 3$.

Enter the functions into GGB.



a. Find the points of intersection.

Command: intersect [f, g]

Answer: −1.5341
0.4827
4.0514

b. Set-up the integrals needed to calculate the desired area.

$$\int_{-1.5341}^{0.4827} (f - g)\, dx + \int_{0.4827}^{4.0514} (g - f)\, dx$$

c. Calculate the area.

Command: integral between [f, g, −1.5341, 0.4827] + integral between [g, f, 0.4827, 4.0514]

Answer: 35.0501

24. Suppose the demand function for a product is x thousand units per week and the corresponding wholesale price, in dollars, is $D(x) = \sqrt{174 - 8x}$. Determine the consumers' surplus if the wholesale market price is set at $8 per unit.

Recall: $CS = \int_0^{Q_E} D(x)\,dx - \underline{Q_E \cdot P_E}$

$PS = Q_E \cdot P_E - \int_0^{Q_E} S(x)\,dx$

a. Find the quantity demanded.

Enter the demand function into GGB. Then create a constant function f(x) = 8, in order to find the point of intersection of the demand function and the constant function. This answer will the ***quantity sold.***

Command: intersect [D, f]

Answer: $(\underline{13.75}, 8)$

b. Find the consumers' surplus if the market price for the product is $8 per unit.

Recall: $CS = \int_0^{Q_E} D(x)\,dx - Q_E \cdot P_E$

First apply the formula.

$$\int_0^{13.75} \sqrt{174 - 8x}\, dx - 13.75 * 8$$

Command: Answer:

integral[D, 0, 13.75] - 13.75*8

= 38.60

25. Let $f(x,y) = 6x^3 - 5x^2 y + 4xy^2 + 8y^3 + 3$.

a. Find $f(1, 1)$.

Command: Answer:

f(1,1)

= 16

b. Find the first order partial derivatives.

Commands: Answer:

derivative[f, x] $f_x = a(x,y) = 18x^2 - 10xy + 4y^2$

derivative[f, y] $f_y = b(x,y) = -5x^2 + 8xy + 24y^2$

c. Find the second order partial derivatives.

Commands: Answer:

derivative[a, x] $f_{xx} = 36x - 10y$

derivative[a, y] $f_{xy} = -10x + 8y$

derivative[b, x] $f_{yx} = -10x + 8y$

derivative[b, y] $f_{yy} = 8x + 48y$

For the next problem, recall:

$D(x,y) = f_{xx} f_{yy} - (f_{xy})^2$

- If $D(a, b) > 0$ and $f_{xx}(a,b) < 0$, then f has a <u>relative maximum</u> at (a, b).
- If $D(a, b) > 0$ and $f_{xx}(a,b) > 0$, then f has a <u>relative minimum</u> at (a, b).
- If $D(a, b) < 0$, then f has neither a relative maximum nor a relative minimum at (a, b) (i.e., it has a saddle point, which is neither a max nor a min).
- If $D(a, b) = 0$, then this test is inconclusive.

26. Use the second derivative test to find the relative extrema of $f(x, y) = 5x^3 - 2xy + 6y^2$.
Enter the function into GGB.

a. Find the first-order partials.

Command: derivative[f, x]
Answer: $f_x = a(x,y) = 15x^2 - 2y$

Command: derivative[f, y]
Answer: $f_y = b(x,y) = -2x + 12y$

b. Set each first-order partial equal to zero and enter each into GGB.

Command: $15x^2 - 2y = 0$
Answer: c: $15x^2 - 2y = 0$

Command: $-2x + 12y = 0$
Answer: d: $-x + 6y = 0$

c. Find the point of intersection of the equations in part b. These points of intersection are the critical points of the function *f*.

Command: intersect [c, d]
Answer: $(0, 0)$

$(0.0222, 0.0037)$

d. Determine $D(x, y) = f_{xx} \cdot f_{yy} - \left(f_{xy}\right)^2$.

Command: derivative[a, x]
Answer: $f_{xx} = 30x$

Command: derivative [b, y]
Answer: $f_{yy} = 12$

Command: derivative [a, y]
Answer: $f_{xy} = 2$

$D = 30x(12) - (2)^2 = 360x - 4$

e. Apply the second derivative test to classify each critical point found in step C.

$D(0,0) = 360(0) - 4$
$= -4 < 0$
Saddle Point

$D(0.0222, 0.0037) = 360(0.0222) - 4 > 0$
$f_{xx}(0.0222, 0.0037) = 30(0.0222) > 0$
R. Min

f. For any maxima point and minima point found in part E, calculate the maxima and minima, respectively.

Command: $f(0.0222, 0.0037)$
Answer: 0.000027