Math 1314
Lesson 18
Finding Indefinite and Definite Integrals

Working with Riemann sums can be quite time consuming, and at best we get a good approximation. In an area problem, we want an exact area, not an approximation. The definite integral will give us the exact area, so we need to see how we can find this.

We need to start by finding an antiderivative:

Antiderivatives (Indefinite Integrals)

Definition: A function F is an antiderivative of f on interval I if $F'(x) = f(x)$ for all x in I.

You can think of an antiderivative problem as asking you to find the *problem* if you are given the *derivative*. Look at what you are given in your problem and ask: "If this is the answer, what was the problem whose derivative I wanted to find?"

Antiderivative problems will use this notation: $\int f(x)\,dx = F(x) + C$, C is an arbitrary constant

Notation: We will use the integral sign $\int$ to indicate integration (antidifferentiation). This indicates that the indefinite integral of $f(x)$ with respect to the variable x is $F(x)+C$ where $F(x)$ is an antiderivative of f.

The reason for "+ C" is illustrated below:

Each function that follows is an antiderivative of $10x$ since the derivative of each is $10x$.
$F(x)=5x^2-1$, $G(x)=5x^2+1$, $H(x)=5x^2+2$, $K(x)=5x^2+2$, etc.

$\int_a^b \quad dx$

Basic Rules

Rule 1: The Indefinite Integral of a Constant *k*

$\int k\,dx = kx + C$

Example 1:

a. $\int -10\,dx$

$= -10x + C$

b. $\int \sqrt{5}\,dx$

$= \sqrt{5}x + C$

Rule 2: The Power Rule

$\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C$, n is a real number with $n \neq -1$

Example 2: $\int x^4 \, dx$

$\dfrac{x^{4+1}}{4+1} = \boxed{\dfrac{x^5}{5} + C}$

Example 3: $\int x^{-7} \, dx$

$\dfrac{x^{-7+1}}{-7+1} = \dfrac{x^{-6}}{-6} + C$

$= \boxed{\dfrac{-1}{6x^6} + C}$

Example 4: $\int \sqrt{x} \, dx$

$= \int x^{1/2} dx$

$\dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \dfrac{x^{\frac{5}{2}}}{3/2} = \boxed{\dfrac{2}{3} x^{3/2} + C}$

Example 5: $\int \dfrac{1}{x^4} dx$

$\int x^{-4} dx$

$= \dfrac{x^{-4+1}}{-4+1} = \dfrac{x^{-3}}{-3} = \boxed{\dfrac{-1}{3x^3} + C}$

Rule 3: The Indefinite Integral of a Constant Multiple of a Function

$\int cf(x)dx = c\int f(x)dx$

Example 6: $\int -5x^9 dx = -5 \int x^9 dx$

$-5 \cdot \dfrac{x^{9+1}}{9+1} = \dfrac{-5x^{10}}{10}$

$= \boxed{-\dfrac{1}{2} x^{10} + C}$

Example 7: $\int 2x^{-5} dx = 2\int x^{-5} dx$

$2 \cdot \dfrac{x^{-5+1}}{-5+1} = \dfrac{2x^{-4}}{-4}$

$= \boxed{\dfrac{-1}{2x^4} + C}$

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Rule 4: The Sum (Difference) Rule

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

Example 8: $\int (2x^2 - 10x - 3)dx$

$$\frac{2x^{2+1}}{2+1} - \frac{10x^{1+1}}{1+1} - 3x$$

$$\boxed{\frac{2}{3}x^3 - 5x^2 - 3x + C}$$

Example 9: $\int (11x^{10} - 4x^9 + 1)dx$

$$\frac{11x^{10+1}}{10+1} - \frac{4x^{9+1}}{9+1} + 1x$$

$$\boxed{x^{11} - \frac{2}{5}x^{10} + x + C}$$

The Fundamental Theorem of Calculus

Finding the antiderivative is a tool that we need in order to find the definite integral of a function over an interval. Next we apply **the fundamental theorem of calculus**:

Top - Bottom

Let f be a continuous function on $[a, b]$. Then $\int_a^b f(x)dx = F(b) - F(a)$ where $F(x)$ is any antiderivative of f.

This says that we can find the definite integral by first finding the antiderivative of the function that's given and then by evaluating the antiderivative at the upper and lower limits of integration and subtracting.

Properties of Definite Integrals

Suppose $f(x)$ and $g(x)$ are integrable functions. Then:

1. $\int_a^a f(x)\,dx = 0$
2. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$
3. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
4. $\int_a^b [f(x) \pm g(x)]\,dx = \int_b^a f(x)\,dx \pm \int_a^b g(x)\,dx$
5. $\int_a^b f(x)\,dx = \int_c^a f(x)\,dx + \int_c^b f(x)\,dx, \quad a < c < b$

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1. **Integrate** $f(x) = 17$ $\int 17\,dx$

   A. 17 + C **B.** 17 x +C **C.** 0 + C **D.** 1 +C

2. **Integrate** $f(x) = 4x^3$ $\int 4x^3\,dx$

   A. $12x^2$ +C **B.** $4x^4$ + C **C.** x^4 + C **D.** x^3 + C

Example 10: Evaluate: $\int_{-1}^{5}(3x^2-6)dx$

Recall: $\int_a^b [f(x) \pm g(x)]\,dx = \int_b^a f(x)\,dx \pm \int_a^b g(x)\,dx$

$\frac{3x^{2+1}}{2+1} - 6x = \underbrace{x^3 - 6x}_{\text{Antiderivative}} + C$

$F(5) - F(-1)$

$((5)^3 - 6(5)) - ((-1)^3 - 6(-1))$

$(125-30) - (-1+6)$

$(95) - (5) = \boxed{90}$

integral [f] Indefinite Integrals

$\int_a^b f(x)\,dx$ in GGB integral[f, a, b]

Q.4 is C

Q_3 is A