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# Math 1314
## Lesson 19
## Numerical Integration

$\int_a^b f(x)dx$

For more complicated functions, we will use GeoGebra to find the definite integral.

The command is: integral[<Function>, <Start x-Value>, <End x-Value>]

The "<Start x-Value>" is the lower limit of integration and the "<End x-Value>" is the upper limit of integration.

***Begin each problem by entering the function into GGB.***

Example 1: Evaluate: $\int_1^4 \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ ← f(x)

Command: Answer:

integral[f, 1, 4] = 0.6363

Example 2: Evaluate $\int_3^4 \frac{18.6}{1+1.2e^{-4x}} dx$

Command: Answer:

integral[f, 3, 4] = 18.6

Example 3: Evaluate $\int_1^2 \frac{x^2}{3x^3+6} dx$

Command: Answer:

integral[f, 1, 2] = 0.1338

We can use integration to solve a variety of problems.

Suppose we are given the rate at which a worker can produce items along an assembly line. This is a derivative – a rate of change. When we find the antiderivative, we are finding a function that gives us the total number of items that can be produced. So in this instance, the area under the curve will give total production.

Example 4: A study of worker productivity shows that the rate at which a typical worker can produce widgets on an assembly line can be expressed as $N(t) = -3t^2 + 12t + 15$ where $t$ gives the number of hours after a worker's shift has begun. In GGB make X

a. Set up the integral that represents the number of widgets a worker can produce during the last hour of a five hour shift.

$$\int_4^5 (-3t^2 + 12t + 15)\, dt$$

b. Determine the number of widgets a worker can produce during the last hour of a five hour shift.

Command: integral[N, 4, 5]

Answer: 8

The velocity function is the derivative of the position function. When we find the antiderivative of the velocity function, we have the position function, and the area under the curve will give the total distance traveled.

Example 5: Suppose you are driving a car and that your velocity can be approximated by $v(t) = 2t\sqrt{25 - t^2}$, where $t$ is measured in seconds and $v$ is measured in feet per second.

Make "t" an "x"

a. Set up the integral that represents the distance traveled by the car in the first 5 seconds.

$$\int_0^5 2t\sqrt{25-t^2}\, dt$$

b. How far will you travel in the 5 seconds from $t = 0$ to $t = 5$?

Command: integral[v, 0, 5]

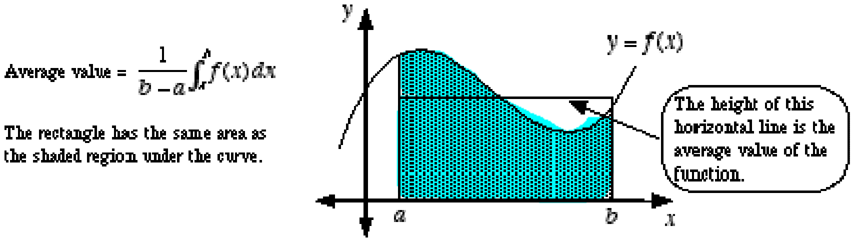
Answer: $83.\bar{3}$ ft

So, when you "go backwards" from a function that gives a rate of change, you get a function that gives the total quantity for that function. Then the area under the curve is the total quantity for that function between two given numbers, $a$ and $b$.

## The Average Value of a Function

We can use the definite integral to find the average value of a function.

Suppose $f$ is an integrable function on the interval $[a, b]$. Then the average value of $f$ over the interval is $\frac{1}{b-a}\int_a^b f(x)dx$. This is what average value represents:




Example 6: Find the average value of $f(x) = x^2 - 3x + 5$ on [2, 5].

a. Set up the integral that represents the average value of the function over the specified interval.

*Recall:* $\frac{1}{b-a}\int_a^b f(x)dx$ $\quad \frac{1}{5-2}\int_2^5 f(x)dx = \frac{1}{3}\cdot\int_2^5 f(x)dx$

b. Determine the average value.

Command: $\frac{1}{3}$ * integral[f, 2, 5]

Answer: = 7.5

Example 7: The sales of ABC Company in the first $t$ years of its operation is approximated by the function $S(t) = t\sqrt{0.4t^2 + 4}$ where $S(t)$ is measured in millions of dollars. What were the company's average annual sales over its first five years of operation? $0 \to 5$

a. Set up the integral that represents the average value of the function over the specified interval.

*Recall:* $\frac{1}{b-a}\int_a^b f(x)dx$ $\quad \frac{1}{5-0}\int_0^5 f(x)dx$

b. Determine the average value.

Command: 1/5 * integral[f, 0, 5]

Answer: = 7.3972 millions of dollars

or

$7,397,200

# Math 1314
# Lesson 20
# Area Between Two Curves

The general "formula" for computing the area between two curves is $\int_a^b (\text{top function - bottom function})dx$.

Command: integralbetween[<function>, <function>, <start $x$-value>, <end $x$-value>]

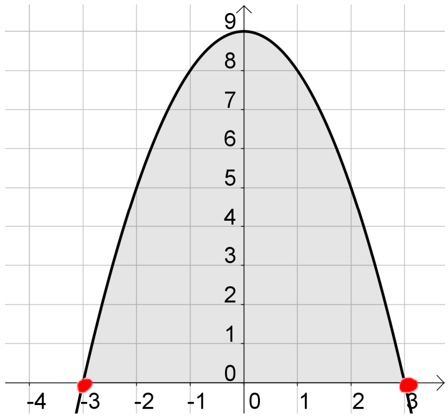
Top, Bottom, a, b

Example 1: The graph of $f(x) = -x^2 + 9$ is given below. Find the area of the shaded region.

x-axis : y = 0
In GGB
g(x) = 0

a. Set-up the integral needed to calculate the desired area.

$$\int_{-3}^{3} (-x^2+9 - 0)\,dx$$




b. Calculate the area.

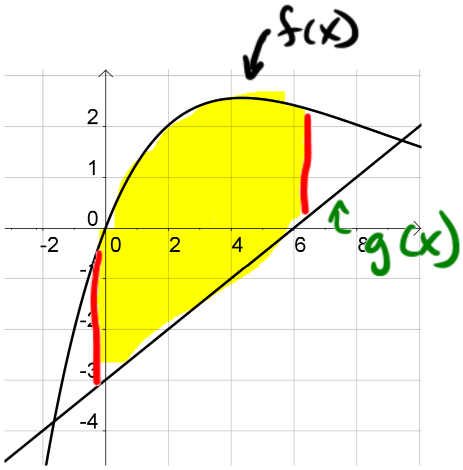
integral between[f, g, -3, 3]

= 36

Example 2: Find the area between $f(x) = 1.6xe^{-0.23x}$ and $g(x) = 0.5x - 3$ on the interval $-0.07 \le x \le 6.46$.
*Enter the functions into GGB.*

a. Set-up the integrals needed to calculate the desired area.

$$\int_{-0.07}^{6.46} (1.6x\,e^{-0.23x} - (0.5x - 3))\,dx$$




b. Calculate the area.

integralbetween[f, g, -0.07, 6.46]

= 22.3842



Popper 16

1. A. True $\boxed{\text{B. False}}$

2. A 5 B 6 C 8 D 10

3. A 1 B. 4. C. 7 D 12

4 A. 1 B 68 C. 73. D. 63

5 E

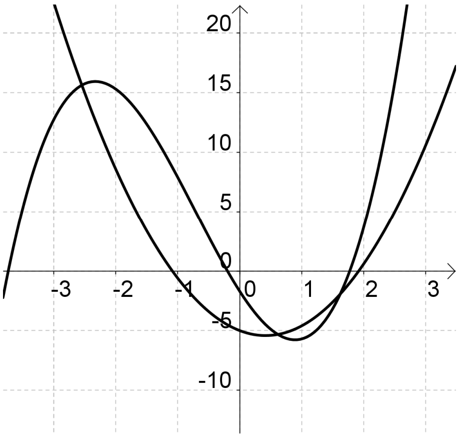
Z

Sometimes the limits of integration are not given. In this case, we'll need to graph the functions and find the point(s) of intersection of the functions. In GGB we'll use either command:

intersect[<object>, <object>] or intersect[<function>, <function>, <start *x*-value>, <end *x*-value>]

Example 3: Find the area completely enclosed by the graphs of the functions $f(x) = 1.3x^3 + 2.8x^2 - 8.1x - 1.7$ and $g(x) = 2.4x^2 - 2x - 5$.

*Enter the functions into GGB.*




a. Find the points of intersection.

Command: Answer:

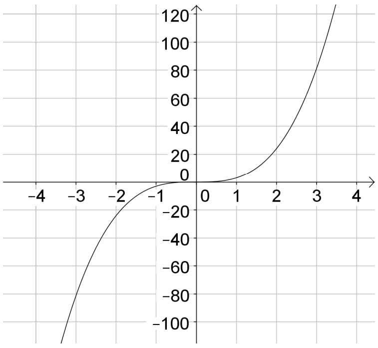
b. Set-up the integrals needed to calculate the desired area.

c. Calculate the area.

Command: Answer:

Example 4: Find the area of the region that is completely enclosed by the graph of $f(x) = 3x^3$ from $x$ = -3 to $x$ = 3.

*Enter the function into GGB.* a. Set-up the integral(s) needed to calculate the desired area.

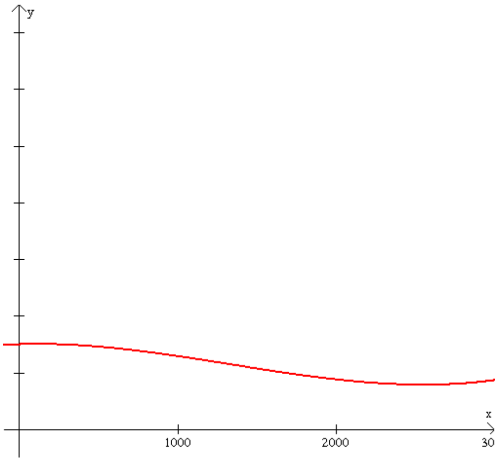



b. Calculate the area.
Command: Answer:

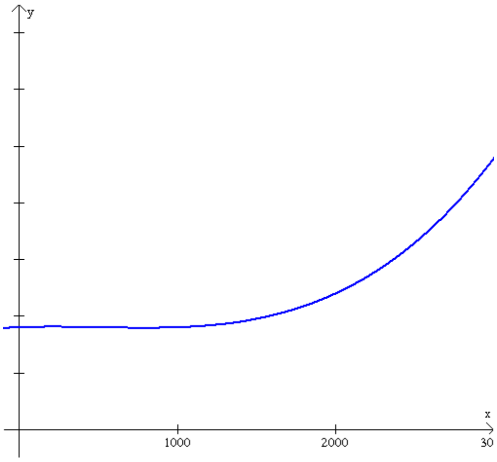
Now let's look at some applications.

Two advertising agencies are competing for a major client. The rate of change of the client's revenues using Agency A's ad campaign is approximated by $f(x)$ below. The rate of change of the client's revenues using Agency B's ad campaign is approximated by $g(x)$ below. In both cases, $x$ represents the amount spent on advertising in thousands of dollars. In each case, total revenue is the area under the curve given in thousands of dollars.

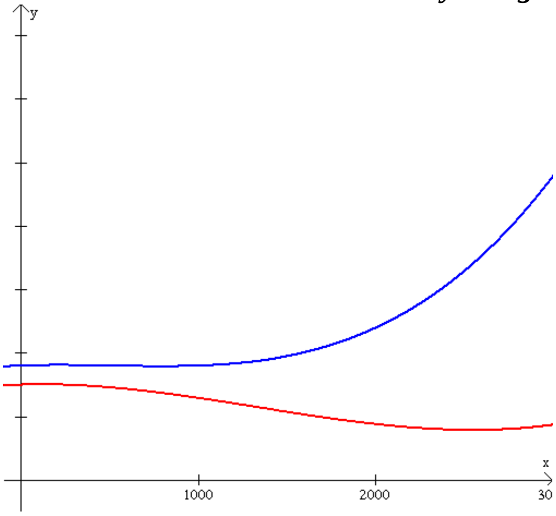
Agency A




Agency B



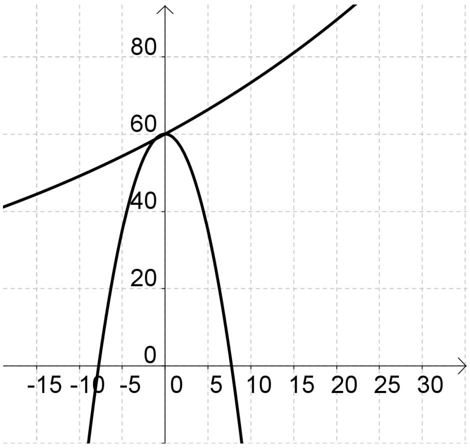
Now let's put the two graph on the same grid. The graph below shows the relationship between the two revenue functions. We see that one function is above the other. ***The area between the two functions represents the projected additional revenue that would be realized by using Agency B's ad campaign.***




This is an example of the kinds of problems you will be able to solve with the techniques you learn in this lesson.

Example 5: Without any effort to curb population growth, a government estimates that its population will grow at the rate of $f(t) = 60e^{0.02t}$ thousand people per year. However, they believe that an education program will alter the growth rate to $g(t) = -t^2 + 60$ thousand people per year over the next 5 years. How many fewer people would there be in the country if the education program is implemented and is successful?

*Enter the functions into GGB.*




Calculate the area.

Command: Answer:

Try this one: The management of a hotel chain expects its profits to grow at the rate of $f(t) = 1 + t^{\frac{2}{3}}$ million dollars per year $t$ years from now. If they renovate some of their existing hotels and acquire some new ones, their profits would grow at the rate of $g(t) = t - 2\sqrt{t} + 4$ million dollars per year. Find the additional profits the company could expect over the next ten years if they proceed with their renovation and acquisition plans.

*Enter the functions into GGB.*