Math 1314 Lesson 20 Area Between Two Curves



36

Example 2: Find the area between $f(x) = 1.6xe^{-0.23x}$ and g(x) = 0.5x - 3 on the interval $-0.07 \le x \le 6.46$. Enter the functions into GGB.

a. Set-up the integrals needed to calculate the desired area.

b. Calculate the area.

integral botween [f, g, -0.07, w.46]

= 22,3842

Lesson 20 - Area Between Two Curves

Sometimes the limits of integration are not given. In this case, we'll need to graph the functions and find the point(s) of intersection of the functions. In GGB we'll use either command:

intersect[<object>, <object>] or intersect[<function>, <function>, <start x-value>, <end x-value>]

Example 3: Find the area completely enclosed by the graphs of the functions $f(x) = 1.3x^3 + 2.8x^2 - 8.1x - 1.7$ and $g(x) = 2.4x^2 - 2x - 5$.



a. Find the points of intersection. Command:

Answer:

intersect [f.g] x= -2.5442, 0.6155, 1.621

b. Set-up the integrals needed to calculate the desired area.



c. Calculate the area. Command:

Lesson 20 - Area Between Two Curves

Answer:

Example 4: Find the area of the region that is completely enclosed by the graph of $f(x) = 3x^3$ from x = -3 to x = 3.

Enter the function into GGB. a. Set-up the integral(s) needed to calculate the desired area.



Now let's look at some applications.

Two advertising agencies are competing for a major client. The rate of change of the client's revenues using Agency A's ad campaign is approximated by f(x) below. The rate of change of the client's revenues using Agency B's ad campaign is approximated by g(x) below. In both cases, x represents the amount spent on advertising in thousands of dollars. In each case, total revenue is the area under the curve given in thousands of dollars.



Lesson 20 - Area Between Two Curves

Now let's put the two graph on the same grid. The graph below shows the relationship between the two revenue functions. We see that one function is above the other. *The area between the two functions represents the projected additional revenue that would be realized by using Agency B's ad campaign*.



This is an example of the kinds of problems you will be able to solve with the techniques you learn in this lesson.

Example 5: Without any effort to curb population growth, a government estimates that its population will grow at the rate of $f(t) = 60e^{0.02t}$ thousand people per year. However, they believe that an education program will alter the growth rate to $g(t) = -t^2 + 60$ thousand people per year over the next 5 years. How many fewer people would there be in the country if the education program is implemented and is successful?



intersect [f,g, -5,5]

Calculate the area.

Command:

integralbetween [f,g,o,s]

Answer:

57.1794

57, 179.4 people

Lesson 20 – Area Between Two Curves

Try this one: The management of a hotel chain expects its profits to grow at the rate of $f(t) = 1 + t^{\frac{2}{3}}$ million dollars per year *t* years from now. If they renovate some of their existing hotels and acquire some new ones, their profits would grow at the rate of $g(t) = t - 2\sqrt{t} + 4$ million dollars per year. Find the additional profits the company could expect over the next ten years if they proceed with their renovation and acquisition plans.



۱

Math 1314 Lesson21 Other Applications of Integration

Consumers' Surplus and Producers' Surplus

The **consumers' surplus** is defined to be the difference between what customers would be willing to pay and what they actually pay. It is the area of the region bounded below by the demand function and above by the line that represents the unit market price. In the sketch shown below, the shaded region represents the consumers' surplus.



represents the demand function, Q_E represents the quantity sold and P_E represents the price.

Similarly, producers may be willing to sell their product for a lower price than the prevailing market price. If the market price is higher than where producers expect to price their items, then the difference is called the **producers' surplus**. The producer's surplus is the area of the region bounded below by the line that represents the price and above by the supply curve. In the sketch shown below, the shaded region represents the producers' surplus.



The producers' surplus is given by $PS = Q_E \cdot P_E - \int_0^{Q_E} S(x) dx$. In this formula, S(x) represents the supply function, P_E represents the unit market price and Q_E represents the quantity supplied.

Example 1: Suppose the demand for a certain product is given by

 $p = D(x) = -0.01x^2 - 0.1x + 6$ where p is the unit price given in dollars and x is the quantity demanded per month given in units of 1000. The market price for the product is \$4 per unit.

a. Find the quantity demanded.

Enter the demand function into GGB. Then create a constant function f(x) = 4, in order to find the point of intersection of the demand function and the constant function. This answer will the *quantity sold*.

Command:

intersect [D, 5]



b. Find the consumers' surplus if the market price for the product is \$4 per unit.

Recall: $CS = \int_0^{Q_E} D(x) dx - Q_E \cdot P_E$

First apply the formula.

$$\int_{0}^{10} D(x) dx - 10 + 4$$

Command:

Answer:

Lesson 21 - Other Applications of Integration

Sometimes, the unit price will not be given. Instead, product will be sold at market price, and you'll be given both supply and demand equations. In this case, we can find the equilibrium point (Section 4.2) which will give us the equilibrium quantity and price.

Example 2: The demand function for a popular make of 12-speed bicycle is given by $p = D(x) = -0.001x^2 + 250$ where p is the unit price in dollars and x is the quantity demanded in units of a thousand. The supply function for the same product is given by $p = S(x) = 0.0006x^2 + 0.02x + 100$ where p is the unit price in dollars and x is the quantity supplied in units of a thousand.

Qe

Begin by entering the functions into GGB.

a. Assume the market price is set at the equilibrium price. Find the equilibrium point. Command: Answer:

intersect [D, S]

(-312 5362, 52, 2939) (249.9612, 160.0233)

~ (300,160)

Recall: $CS = \int_{0}^{Q_E} D(x) dx - Q_E \cdot P_E$

First apply the formula.

D(x) dx - 300 x 160

Command:

Answer:

Lesson 21 – Other Applications of Integration

c. Determine the producers' surplus.

Recall: $PS = Q_E \cdot P_E - \int_0^{Q_E} S(x) dx$

First apply the formula.

Command:

Answer:

```
00 , 00 , 71
```

X (000

Probability

The study of probability deals with the likelihood of a certain outcome of an experiment. When you flip a coin, the probability that the coin lands with heads facing up is $\frac{1}{2}$. When you roll a six-sided die and record the number that lands on the uppermost face, the probability that the die lands with the number 3 facing upwards is $\frac{1}{6}$.

You can also find the probability of something occurring over a continuous interval. Suppose you want to know how long a brand of light bulb lasts. Now the possible set of answers contains more than a discrete set of numbers (e.g., something other than 1, 2, 3, etc.). The light bulbs can last any positive length of time. We can find the probability that the light bulbs life span is on a given interval [a, b].

This problem is an example of a problem that involves a probability density function. This type of function can be used to determine the probability that an event occurs on a given interval [a, b]. All probability density functions must meet these three criteria:

- 1. $f(x) \ge 0$ for all x
- 2. The area under the graph of f(x) is exactly 1.

3. The probability that an event occurs in an event [a, b] can be computed using the definite integral $\int_{a}^{b} f(x) dx$.

Lesson 21 - Other Applications of Integration

Example 3: The function $f(x) = 0.002e^{-0.002x}$ gives the life span of a popular brand of light bulb, where x gives the lifespan in hours and f(x) is the probability density function. Find the probability that the lifespan is between 500 hours and 1000 hours.

a. Set up the integral needed to answer the question.

$$\int_{-0.002e}^{0.002e} 0.002e^{-0.002e} h_{r}$$

b. Find the probability. Command:

Answer:

= 0.2325

integral [f, 500,1000]

Example 4: A company finds that the percent of its locations that experience a profit in the first year of business has the probability density function $P(x) = \frac{36}{11}x\left(1-\frac{1}{3}x\right)^2$, $0 \le x \le 1$.

What is the probability that more than 50% of the company's locations experienced a profit during the first year of business?

a. Set up the integral needed to answer the question.

$$\int_{0}^{1} \frac{36}{11} \times \left(1 - \frac{1}{3} \times\right)^{2} dx$$

b. Find the probability.Command:

integral [P, o.s, 1]

Answer:

0.6761

3.E



Lesson 21 – Other Applications of Integration