## Lesson 22 Functions of Several Variables

So far, we have looked at functions of a single variable. In this section, we will consider functions of more than one variable. You are already familiar with some examples of these.

$$P(x, y) = 2x + 2y$$
$$A(P, i, t) = P(1+i)^{t}$$

These formulas are functions of several variables. We have just never called them that before. We will, for the most part, limit our discussion to functions of two variables.

## **Functions of Two Variables**

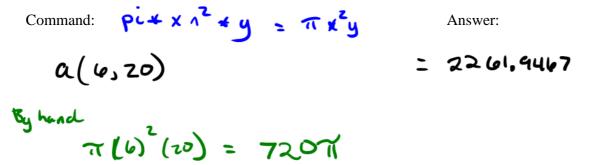
**Definition**: A real valued function of two variables, f, consists of a set A of ordered pairs of real numbers (x, y) called the domain of the function, and a rule that associates with each ordered pair in the domain of f one and only one real number, denoted by z = f(x, y).

You will need to learn two skills using functions of several variables: Evaluating at a given point and determining the domain.

Example 1: Suppose  $f(x, y) = 3x^2y - 6 + \ln(xy)$  Compute f(-1, -3). Enter the function as shown in GGB.

Command:  $3 \times \wedge 2 \times y - 6 + \ln(2 \times y) \Rightarrow a(x,y)$  a(-1, -3) = -3,9014 $3(-1)^{2}(-3) - 6 + \ln(-1, -3) = -9 - 6 + \ln(3) = -15 + \ln(3)$ 

Example 2: The volume of a cylindrical tank with radius *r* and height *h* is given by the formula  $f(r,h) = \pi r^2 h$ . Find the volume of a tank with radius 6 feet and height 20 feet. *Enter the function as shown in GGB.* 



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Example 3: The monthly payment that amortizes a loan of A dollars in t years when the interest rate is *r* per year is given by

$$P = f(A, r, t) = \frac{Ar}{\left(12\left(1 - \left(1 + \frac{1}{12}r\right)^{-12t}\right)\right)}$$

Find the monthly payment for a mortgage of \$250,000 that will be amortized over 25 years with an interest rate of 4.5% per year. 0.045 Enter the function as shown in GGB.

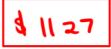
Command:

years.

Answer:

$$P(a50,000, 0.045, 25) = [389.54]$$
  
Example 4: Use the following table to answer the following question.  
Find  $P = f(A, r, t) = f(150, 5.25, 20)$ , where *P* is the monthly payment, *A* is the amount financed in thousands of dollars, *r* is the interest rate, and *t* is the length of the loan in

Monthly Payments in Dollars at 5.25% APR					
Amt	10	15	20	25	30
Financed	yrs	yrs	yrs	yrs	yrs
\$50,000	618	445	376	329	300
\$75,000	928	683	564	494	449
\$100,000	1237	911	751	659	599
\$125,000	1546	1138	939	823	749
\$150,000	1855	1366	1127	988	899
\$175,000	2164	1594	1315	1153	1049
\$200,000	2475	1821	1503	1317	1198



Example 5: Find the domain of the function: 
$$f(x, y) = \frac{5x}{2x-5}$$

2x- 5y = ∂ -5y =-2x y ≠ = x

Domain = 
$$\left\{ (x,y) \right\} y \neq \frac{2}{5} \times \frac{2}{5}$$

Example 6: Find the domain of the function:  $f(x, y) = \sqrt{16x - y}$ .

Example 7: Find the domain of the function:  $f(x, y) = \frac{2x^2 + 3y^2}{2x^2 + 3y^2}$ 

Polynomials D: {(X,y) | All real numbers }

