**Lesson 22**
**Functions of Several Variables**

So far, we have looked at functions of a single variable. In this section, we will consider functions of more than one variable. You are already familiar with some examples of these.

$$P(x, y) = 2x + 2y$$
$$A(P, i, t) = P(1 + i)^t$$

These formulas are functions of several variables. We have just never called them that before. We will, for the most part, limit our discussion to functions of two variables.

## Functions of Two Variables

**Definition**: A real valued function of two variables, $f$, consists of a set $A$ of ordered pairs of real numbers $(x, y)$ called the domain of the function, and a rule that associates with each ordered pair in the domain of $f$ one and only one real number, denoted by $z = f(x, y)$.

You will need to learn two skills using functions of several variables: Evaluating at a given point and determining the domain.

Example 1: Suppose $f(x, y) = 3x^2y - 6 + \ln(xy)$ Compute $f(-1,-3)$.
*Enter the function as shown in GGB.*

Command: 3x^2*y-6+ln(x*y) => a(x,y) Answer:

a(-1,-3)

= -13.9014

$3(-1)^2(-3) - 6 + \ln(-1 \cdot -3) = -9 - 6 + \ln(3) = -15 + \ln(3)$

Example 2: The volume of a cylindrical tank with radius $r$ and height $h$ is given by the formula $f(r, h) = \pi r^2 h$. Find the volume of a tank with radius 6 feet and height 20 feet.
*Enter the function as shown in GGB.*

Command: pi*x^2*y = $\pi x^2 y$ Answer:

a(6,20)

= 2261.9467

By hand

$\pi(6)^2(20) = 720\pi$

Example 3: The monthly payment that amortizes a loan of $A$ dollars in $t$ years when the interest rate is $r$ per year is given by

$$P = f(A, r, t) = \frac{A r}{\left(12\left(1-\left(1+\frac{1}{12}r\right)^{-12t}\right)\right)}$$

Find the monthly payment for a mortgage of $250,000 that will be amortized over 25 years with an interest rate of 4.5% per year. → 0.045
*Enter the function as shown in GGB.*

Command: Answer:

P(250000, 0.045, 25) = 1389.56

Example 4: Use the following table to answer the following question.
Find $P = f(A,r,t) = f(150, 5.25, 20)$, where $P$ is the monthly payment, $A$ is the amount financed in thousands of dollars, $r$ is the interest rate, and $t$ is the length of the loan in years.

**Monthly Payments in Dollars at 5.25% APR**

| Amt Financed | 10 yrs | 15 yrs | 20 yrs | 25 yrs | 30 yrs |
|---|---|---|---|---|---|
| $50,000 | 618 | 445 | 376 | 329 | 300 |
| $75,000 | 928 | 683 | 564 | 494 | 449 |
| $100,000 | 1237 | 911 | 751 | 659 | 599 |
| $125,000 | 1546 | 1138 | 939 | 823 | 749 |
| $150,000 | 1855 | 1366 | 1127 | 988 | 899 |
| $175,000 | 2164 | 1594 | 1315 | 1153 | 1049 |
| $200,000 | 2475 | 1821 | 1503 | 1317 | 1198 |

$1127

Example 5: Find the domain of the function: $f(x,y) = \frac{3x}{2x-5y}$.

$2x - 5y = 0$

$-5y = -2x$

$y \neq \frac{2}{5}x$

Domain = $\{(x,y) \mid y \neq \frac{2}{5}x\}$

Example 6: Find the domain of the function: $f(x,y)=\sqrt{16x-y}$.

Inside $\geq 0$

$16x-y \geq 0$

$16x \geq y$

$y \leq 16x$

Domain: $\{(x,y) \mid y \leq 16x\}$

Example 7: Find the domain of the function: $f(x,y)=2x^2+3y^2$

Polynomials

D: $\{(x,y) \mid$ All real numbers $\}$

Popper 18

32. B

16 A

47 C

1. D

50. E