Math 1314
Test 3 Review
Material covered is from Lessons 9 – 15

1. The total weekly cost of manufacturing x cameras is given by the cost function: $C(x) = -.03x^2 + 80x + 3000$ and the revenue function is $R(x) = -.02x^2 + 600x$. ← Into GGB

Use the marginal profit function to approximate the actual profit realized on the sale of the 234th unit.

$P(x) = R(x) - C(x)$

$P'(233) = \$524.66$

2. A music company produces a variety of electric guitars. The total cost of producing x guitars is given by the function $C(x) = 6100 + 7x - \frac{1}{5}x^2$ where $C(x)$ is given in dollars. Find the average cost of producing 130 guitars.

Recall: $\overline{C(x)} = \frac{C(x)}{x}$

$f(x) = \frac{C(x)}{x}$

$f(130) = \boxed{27.92}$

Recall:
Demand is said to be **elastic** if $E(p) > 1$.
Demand is said to be **unitary** if $E(p) = 1$.
Demand is said to be **inelastic** if $E(p) < 1$.

3. Suppose $E(p) = \frac{1}{4}$ when the price of the item is p. Then the demand is
a. Elastic b. Unitary c. Inelastic

Solve for x

4. Suppose the demand equation of a product is given by $p = -0.04x + 1000$ where the function gives the unit price in dollars when x units are demanded. Compute $E(p)$ when $p = 535$ and interpret the results.

Recall: $E(p) = -\frac{p \cdot f'(p)}{f(p)}$

$0.04x = -p + 1000$

$x = -25p + 25000 = f(p)$

$f'(p) = -25$

$f'(535) = -25$

$f(535) = 11{,}675$

$E(p) = -\frac{p \cdot f'(p)}{f(p)}$

$= \frac{-535(-25)}{11{,}675}$

$= \boxed{1.15 > 1 \quad \text{Elastic}}$

Popper 13 47. A

5. The sales from company ABC for the years 1998 - 2003 are given below.

	0	1	2	3	4	5
Year	1998	1999	2000	2001	2002	2003
Profits in millions of dollars	36.3	39.1	41.7	44.6	47.9	49.9

> list1

Rescale the data so that $x = 0$ corresponds to 1998.
A. Find an exponential regression model.
Command: Answer:

fitexp[list1]

$\boxed{f(x) = 36.5595e^{0.0644x}}$

B. Find the rate at which the company's sales were changing in 2007. ← $t = 9$
Command: Derivative Answer:

$f'(9) = 4.2101$ millions / yr

6. The number of deer present in a nature preserve can be expressed using the model $N(t) = \frac{125}{1+31e^{-0.6t}}$, where $N(t)$ gives the number of deer and t gives the number of months since the initial count of deer was taken. *Enter the function in GGB.*

A. How many deer will be present after 6 months?
Command: Answer:

N(6) = 67.676 ≈ 68 deer

B. At what rate is the population changing after 6 months?
Command: Answer:

N'(6) = 18.6214 ≈ 19 deer/month

7. . At the beginning of an experiment, a researcher has 511 grams of a substance. If the half-life of the substance is 16 days:

$\frac{511}{2}$

A. Identify two points given in the problem.

(0, 511) (16, 255.5)

B. Find an exponential regression model using the two points in part a.

Command: fitexp[list 1]

Answer: $f(x) = 511e^{-0.0433x}$

C. How many grams of the substance are left after 25 days?

Command: f(25) =

Answer: 173.0061 grams

D. What is the rate of change after 10 days?

Command: f'(10) :

Answer: -14.3593

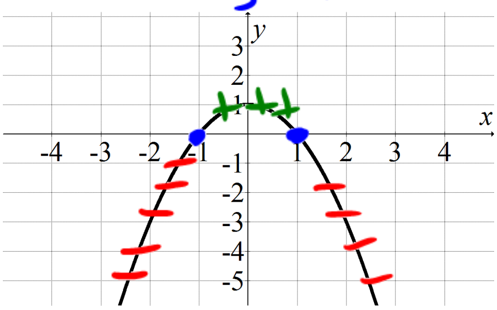
Decreasing at 14.3593 grams / Day

8. The graph given below is the *first derivative* of a function, *f*.

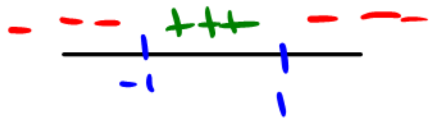
A. Find any critical numbers of *f*.

$f'(x) = 0$ & $f'(x) =$ undefined

$x = -1, 1$



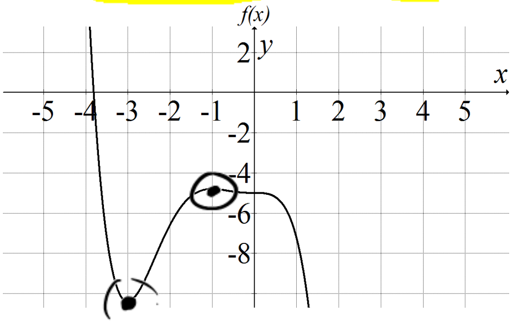
B. Find any intervals where the *f* is increasing/decreasing.



Decreasing: $(-\infty, -1) \cup (1, \infty)$

Increasing: $(-1, 1)$

9. Let $f(x) = -0.2x^5 - x^4 - x^3 - 5$. *Enter the function in GGB.*

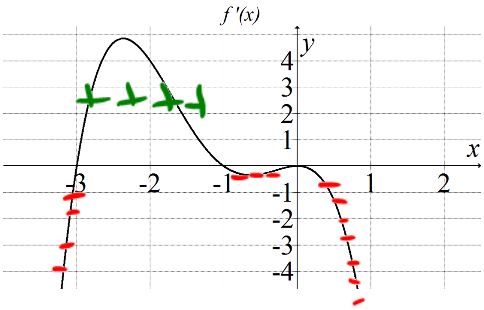


A. Find any critical numbers of *f*.

Command: root [f'(x)]

Answer: (-3, 0) (0, 0) (-1, 0) x = -3, -1, 0

B. Interval(s) on which f is increasing; interval(s) on which f is decreasing.



- - -   + + +   - - - - - - -

-3   -1   0

Increasing: (-3, -1)

Decreasing: (-∞, -3) ∪ (-1, ∞)

C. Coordinates of any relative extrema.

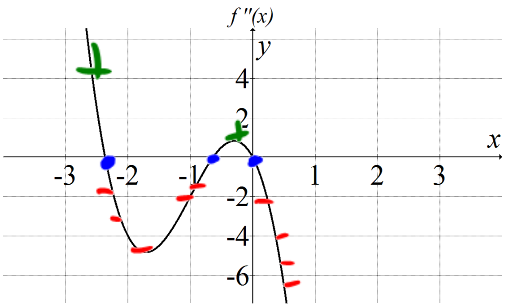
Command: extremum [f]

Answer: (-3, -10.4) R. Min (-1, -4.8) R. Max

D. Interval(s) on which f is concave upward; interval(s) on which f is concave downward.

Command: f''(x)

Command: root [f''(x)]



+ + +   - - -   + + +   - - -

-2.366   -0.634   0

Concave Up: (-∞, -2.366) ∪ (-0.634, 0)

Concave Down: (-2.366, -0.634) ∪ (0, ∞)

E. Coordinates of any inflection points.

Command: inflectionpoint [f]

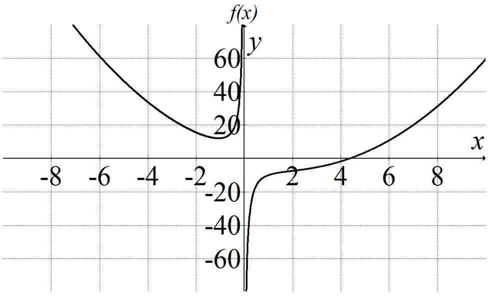
Answer:

(-2.366, -8.2637)

(-0.634, -4.8463)

(0, -5)

10. Let $f(x)=\dfrac{x^3-4x^2-7}{x}$. *Enter the function in GGB.*

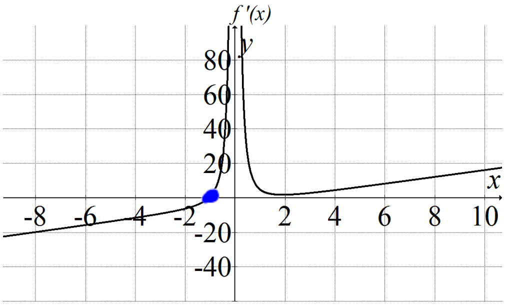


$x \neq 0$

D: $(-\infty, 0) \cup (0, \infty)$

A. Interval(s) on which f is increasing; interval(s) on which f is decreasing.

Command: roots [f'(x), -2, 0]



$x = -1.0681$

-- | +++ | +++

-1.0681 0

Inc: $(-1.0681, 0) \cup (0, \infty)$

Dec: $(-\infty, -1.0681)$

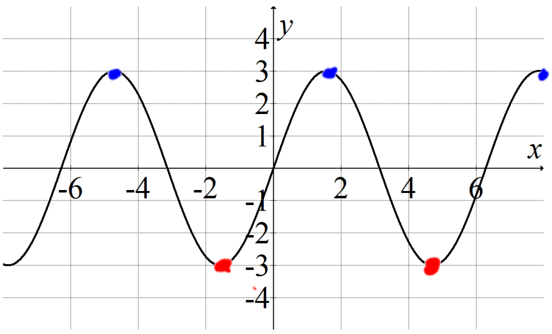
B. Coordinates of any relative extrema.

Command: extremum [f, -2, 0]

Answer: $(-1.0681, 11.9669)$

R. Min

11. Find the absolute maximum and absolute minimum of this function.

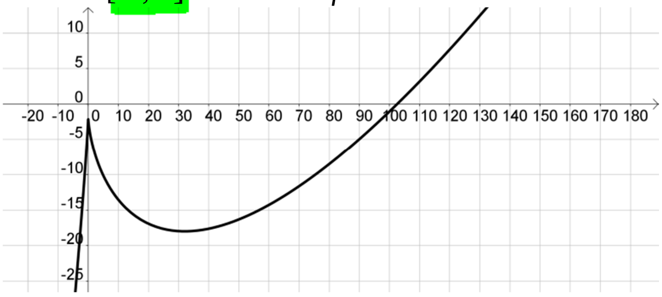


Abs Max = 3

Abs Min = -3

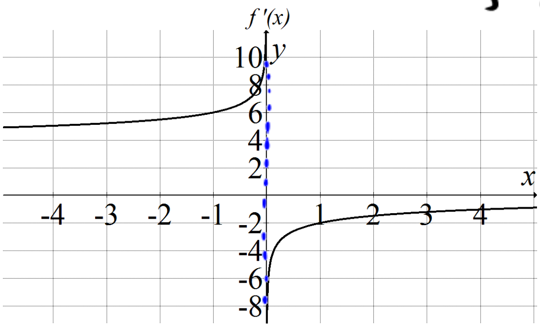
Popper #1 D

12. Find the value of x that gives the absolute maximum value of $f(x) = 2x - 5x^{4/5} - 2$ on the interval [-1, 2]. *Enter the function in GGB.*



No roots b/c $f'(x) \neq 0$

Command: $f'(x)$



Where $f'(x)$ = undefined?

$x = 0$

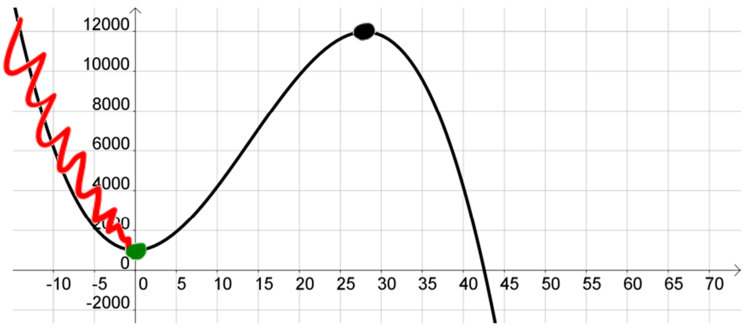
$f(-1) = -9$

$f(0) = -2$

$f(2) = -6.7055$

Abs Max value is -2

13. The mosquito population is a function of rainfall, and can be approximated by the formula $N(x) = 1000 + 42x^2 - x^3$, where x is the number of inches of rainfall. Note that x is non-negative.



Abs Max?

Command: extremum[N]

Answer:

(0, 1000)

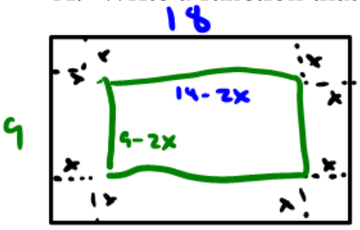
(28, 11976) Abs Max

28 inches of rain

↑ # of Mosquitos

14. An open top box is constructed from a rectangular sheet of material by cutting equal squares from each corner and folding up the flaps. The dimensions of the sheet are 18 inches by 9 inches.

A. Write a function that will give the volume of the box.



$V(x) = x(18-2x)(9-2x)$

B. Find the critical numbers for the function.

root $[V'(x)]$

1.9019

7.0981

C. Find the dimensions of the box with maximum volume.

$V''(1.9019) =$ Neg Value Max

$V''(7.0981) =$ Pos Value Min

$x = 1.9019$

$18-2(1.9019) = 14.1962$

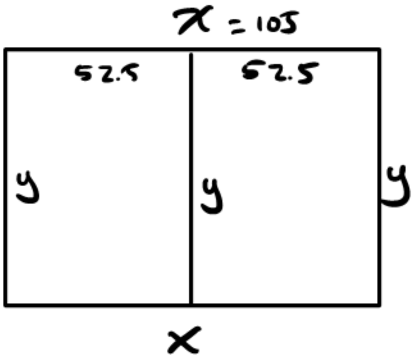
$9-2(1.9019) = 5.1962$

D. Find the maximum volume.

$V(1.9019) = 140.2961$

15. A farmer has 420 feet of fencing to enclose 2 adjacent rectangular pig pens sharing a common side. The two adjacent pens have the same dimensions.

A. Write a function that models the enclosed area.



$2x + 3y = 420$

$3y = 420 - 2x$

$y = 140 - \frac{2}{3}x$

Area $= xy$

$= x(140 - \frac{2}{3}x)$

B. Find the critical numbers for the function.

root $[A'(x)] =$ 105

C. What dimensions should be used for each pig pen so that the enclosed area will be a maximum?

$A''(105) = $ Neg Value (Max)

$y = 140 - \frac{2}{3}(105)$

$= 70$

52.5×70

D. What is the maximum area?

$A = 7350 \text{ ft}^2$

2. E

4. C

3. D