15 MC

Math 1314
Test 4 Review
Lesson 16 – Lesson 24

1. Use Riemann sums with midpoints and 6 subdivisions to approximate the area between $f(x) = \dfrac{45}{1+2e^{-7x}}$ and the x-axis on the interval [1, 9]. a, b

Recall: The command for Riemann Sums is:
RectangleSum[<Function>,<Start x-Value>,<End x-Value>,<Number of rectangles>,<Position for rectangle start>]

"Position of rectangle start": 0 corresponds to left endpoints, 0.5 corresponds to midpoints and
1 corresponds to right endpoints.

Enter the function into GGB.
Command: rectangle sum[f, 1, 9, 6, 0.5]

Answer: 359.999

2. Approximate the area between the curve and the x-axis using upper sums with 50 rectangles on the interval [-2, 2], with $f(x) = x^3 - 7x + 13$

Recall: The command for Upper Sum is:
UpperSum[<Function>,<Start x-Value>,<End x-Value>,<Number of Rectangles>]

The command for Lower Sum is:
LowerSum[<Function>,<Start x-Value>,<End x-Value>,<Number of Rectangles>]

Enter the function into GGB.
Command: UpperSum[f, -2, 2, 50]

Answer: 52.6605

3. Find the indefinite integral $\int \underbrace{(2x^3+5x^2-4x+3)}_{f(x)}\,dx$

Power Rule

$$\frac{2x^{3+1}}{3+1} + \frac{5x^{2+1}}{2+1} - \frac{4x^{1+1}}{1+1} + 3x + C$$

$$\frac{1}{2}x^4 + \frac{5}{3}x^3 - 2x^2 + 3x + C$$

In GGB

integral[f] $= \frac{1}{2}x^4 + \frac{5}{3}x^3 - 2x^2 + 3x + C$

4. Evaluate the following. $\int_{1.5}^{3.1}\left(-2.69x^2+4.27x+2.45\right)dx$

Recall: The command is: integral[<Function>, <Start x-Value>, <End x-Value>]

The "<Start x-Value>" is the lower limit of integration and the "<End x-Value>" is the upper limit of integration.

Enter the function into GGB.

Command: Answer:

integral [f, 1.5, 3.1] = -4.0527

5. Evaluate the following. $\int_{1.3}^{1.5}\frac{6.95x^2}{\sqrt{3.65x-1.95}}\,dx$

Enter the function into GGB.

Command: Answer:

integral [f, 1.3, 1.5] 1.5335

6. A company estimates that the value of its new production equipment depreciates at the rate of $\frac{dv}{dt}=10000(t-9)$ $0\le t\le 9$, where v gives the value of the equipment after t years. Find the total decline in value of the equipment over the first 5 years. [0, 5]

Enter the function into GGB.

a. Setup the integral needed to answer the question.

$$\int_0^5 10000(x-9)\,dx$$

b. Find the total decline in value of the equipment over the first 5 years.

Command: Answer:

integral [f, 0, 5] -325,000

Total decline is $ 325,000

7. The temperature in Marquette, Michigan over a 12 hour period can be modeled by the function where t is measured in hours with $c(t) = -0.05t^3 + 0.01t^2 + 4.5t + 6.2$; $t = 0$ corresponding to the temperature at 12 noon. Find the average temperature during the period from noon until 7 p.m. [0, 7]

Recall: $\frac{1}{b-a}\int_a^b f(x)dx$

Enter the function into GGB.

a. Set-up the integral needed to answer the question.

$$\frac{1}{7-0}\int_0^7 (-0.05x^3 + 0.01x^2 + 4.5x + 6.2)\,dx$$

b. Find the average temperature during the period from noon until 7 p.m.

Command: 1/7 * integral[f, 0, 7]

Answer: 17.8258

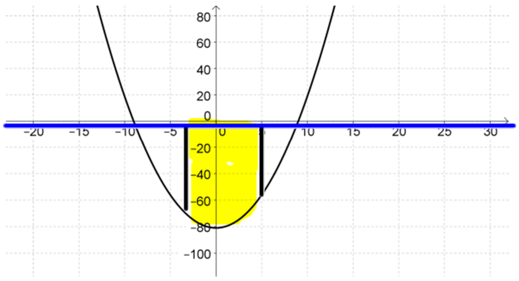
8. Find the area bounded by the graph of $f(x) = x^2 - 81$, the x-axis and the lines $x = -3$ and $x = 5$.

Recall: The general "formula" for computing the area between two curves is

$\int_a^b$ (top function - bottom function)dx. Top Bottom

The command is: IntegralBetween[<Function>, <Function>,<Start x-Value>, <End x-Value>]

Enter the function into GGB.



$g(x) = 0$

a. Set-up the integral needed to calculate the desired area.

$$\int_{-3}^{5} (0 - (x^2 - 81))\,dx = \int_{-3}^{5} (-x^2 + 81)\,dx$$

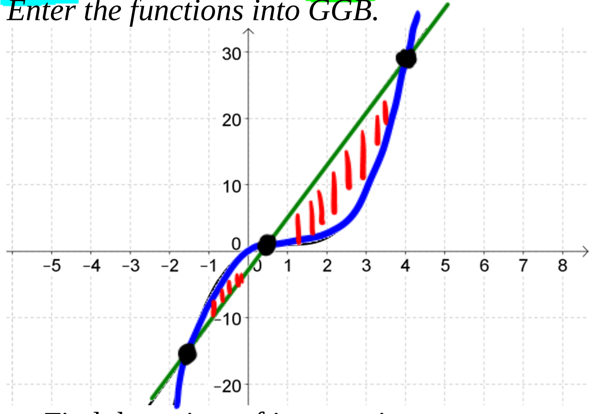
b. Calculate the area.

Command: integralbetween[g, f, -3, 5]

Answer: = 597.3333

9. Find the area of the region(s) that is/are completely enclosed by the graphs of $f(x) = (x-1)^3 + 1$ and $g(x) = 8x - 3$.

Enter the functions into GGB.



a. Find the points of intersection.

Command: intersect[f,g]

Answer: $x = -1.5341$, 0.4427, 4.0514

b. Set-up the integrals needed to calculate the desired area.

Area 1 Area 2

$$\int_{-1.5341}^{0.4427} (f-g)\,dx + \int_{0.4427}^{4.0514} (g-f)\,dx$$

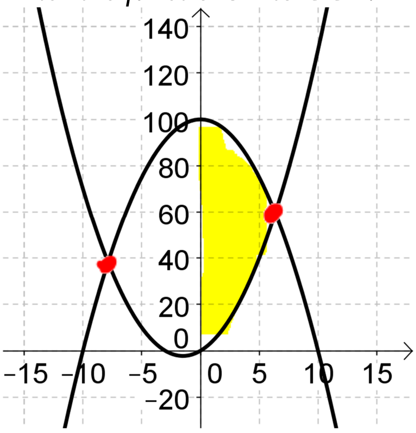
c. Calculate the area.

Command: integralbetween[f,g,-1.5341,0.4427] + integralbetween[g,f,0.4427,4.0514]

Answer: 35.0501

10. A company is considering a new manufacturing process in one of its plants in an effort to save money. The company estimates that the rate of savings can be modeled by $S(x) = 100 - x^2$ where x is time in years and $S(x)$ is given in thousands of dollars per year. At the same time, the company's operating costs will increase, and the company estimates that the rate at which costs will increase can be modeled by $C(x) = x^2 + 3x$ where x is time in years and $C(x)$ is given in thousands of dollars per year. Find the total net savings that the company should expect to realize.

Enter the functions into GGB.



a. Find the points of intersection.

Command: intersect[S, C]

Answer: $x = \cancel{-7.8607}$ 6.3607

b. Set-up the integrals needed to calculate the desired area.

$$\int_0^{6.3607} (S - C)\,dx$$

c. Calculate the area.

Command: integralbetween[S, C, 0, 6.3607]

Answer: = 403.8193

× 1000

\$ 403,819.30

11. Suppose the demand function for a product is x thousand units per week and the corresponding wholesale price, in dollars, is $D(x) = \sqrt{174 - 8x}$. Determine the consumers' surplus if the wholesale market price is set at $8 per unit.

Recall: $CS = \int_0^{Q_E} D(x)\,dx - Q_E \cdot P_E$

$PS = Q_E \cdot P_E - \int_0^{Q_E} S(x)\,dx$

a. Find the quantity demanded.

Enter the demand function into GGB. Then create a constant function $f(x) = 8$*, in order to find the point of intersection of the demand function and the constant function. This answer will the* ***quantity sold.***

Command: intersect[D, f]

Answer: $(13.75, 8)$

Q_E P_E

b. Find the consumers' surplus if the market price for the product is $8 per unit.

Recall: $CS = \int_0^{Q_E} D(x)\,dx - Q_E \cdot P_E$

First apply the formula.

$$\int_0^{13.75} \sqrt{174 - 8x}\,dx - 13.75 * 8$$

Command:

integral[D, 0, 13.75] - 13.75*8

= 38.60

Answer:

x 1000

$\boxed{\$38,600}$

12. The life expectancy of an electrical component has a probability density function that is defined by

$$f(t) = \frac{1}{2} e^{-\frac{1}{2} t}$$

where *t* is given in months after first use. Find the probability that a randomly selected component will last between 6 months and 12 months. [6, 12]

Enter the function into GGB.

a. Set-up the integral needed to answer the question.

$$\int_{6}^{12} \frac{1}{2} e^{-\frac{1}{2}x} \, dx$$

b. Find the probability that a randomly selected component will last between 6 months and 12 months.

Command: Answer:

integral[f, 6, 12] $= 0.0473$

13. Let

$$f(x, y) = \frac{-5.28 x^2 + 4.37 x^2 y + 1.16 x y}{3.94 y^2 + 2.45 x y^3}$$

Find *f* (8, 6).

Enter the function into GGB.

Command: Answer:

f(8,6) 0.319

14. Find the second order partial derivatives of $f(x, y) = 6x^3 - 5x^2 y + 4xy^2 + 8y^3 + 3$

Enter the function into GGB.

Commands: Answer:

derivative [f, x] $f_x = a = 18x^2 - 10xy + 4y^2$

derivative [f, y] $f_y = b = -5x^2 + 8xy + 24y^2$

1st Order Partial

derivative [a, x] $f_{xx} = c = 36x - 10y$

derivative [a, y] $f_{xy} = d = -10x + 8y$

derivative [b, y] $f_{yy} = 8x + 48y$

derivative [b, x] $f_{yx} = -10x + 8y$

2nd Order

15. The productivity of a country is given by the Cobb-Douglas function $f(x, y) = 25x^{0.29}y^{0.71}$, where x represents the utilization of labor and y represents the utilization of capital. If the company uses 1,100 units of labor and 775 units of capital, find the marginal productivity of capital.

Commands: derivative[f, y]

Answer: $f_y = a = \dfrac{\frac{71}{4}x^{0.29}}{y^{0.29}}$

$a(1100, 775)$ = 19.6474 units / units of increase in capital

For the next few problems, recall:

$D(x, y) = f_{xx} f_{yy} - (f_{xy})^2$

- If $D(a, b) > 0$ and $f_{xx}(a,b) < 0$, then f has a relative maximum at (a, b).
- If $D(a, b) > 0$ and $f_{xx}(a,b) > 0$, then f has a relative minimum at (a, b).
- If $D(a, b) < 0$, then f has neither a relative maximum nor a relative minimum at (a, b) (i.e., it has a saddle point, which is neither a max nor a min).
- If $D(a, b) = 0$, then this test is inconclusive.

16. Find the critical points of $f(x, y) = 5x^3 - 2xy + 6y^2$.
Enter the function into GGB.

a. Find the first-order partials.

Command: derivative[f, x]

Answer: $f_x = a = 15x^2 - 2y$

Command: derivative[f, y]

Answer: $f_y = b = -2x + 12y$

Set equal to zero

b. Set each first-order partial equal to zero and enter each into GGB.

Command: $15x^2 - 2y = 0$

Answer: c: $15x^2 - 2y = 0$

Command: $-2x + 12y = 0$

Answer: d: $-x + 6y = 0$

c. Find the point of intersection of the equations in part b. These points of intersection are the critical points of the function f.

Command: intersect[c, d]

Answer: (0, 0)

(0.0222, 0.0037)

17. Suppose

$$f_{xx} = -30x\ ,\ f_{yy} = -6\ ,\ f_{xy} = f_{yx} = 4$$

and the critical points for function f are

$$A = (-0.2869, -0.1913) \text{ and } B = (0.4647, 0.3098)$$

Find the value for D for each critical point and then classify the critical point using the second derivative test.

a. Determine $D(x, y) = f_{xx} \cdot f_{yy} - \left(f_{xy}\right)^2$.

$D(x,y) = -30x \cdot (-6) - (4)^2 = 180x - 16$

b. Apply the second derivative test to classify each critical point.

$D(-0.2869, -0.1913) < 0$

At point A

we have a Saddle Point

$D(0.4647, 0.3098) > 0$

$f_{xx}(0.4647, 0.3098) < 0$

R. Max @ $(0.4647, 0.3098)$

18. Suppose that $f(x, y) = -5x^3 + 7xy - 8y^2$, (0.2042 , 0.0893) is a critical point, $f_{xx}\big|_{(0.2042\ ,\ 0.0893)} = -6.1250$, and $D(0.2042\ ,\ 0.0893) = 49$.

Which of these statements describes the graph of f at (0.2042 , 0.0893) ?

$\left.\begin{array}{l} D > 0 \\ f_{xx} < 0 \end{array}\right\}$ R. Max

a. ~~f has a relative minimum value at f (0.2042 , 0.0893) = 0.0213.~~
b. ~~f has a saddle point at f (0.2042 , 0.0893) = -0.1064.~~
c. f has a relative maximum value at f (0.2042 , 0.0893) = -0.1064.
d. f has a relative maximum value at f (0.2042 , 0.0893) = 0.0213.
e. ~~f has a relative minimum value at f (0.2042 , 0.0893) = -0.1064.~~
f. ~~f has a saddle point at f (0.2042 , 0.0893) = 0.0213.~~

Find $f(0.2042, 0.0893) = 0.0213$

D