Suppose you owned a small business and needed to make some decisions about the pricing of your products. It would be helpful to know what effect a small change in price would have on the demand for your product. If a price change will have no real change on demand for the product, it might make good sense to raise the price. However, if a price increase will cause a big drop in demand, then it may not be a good idea to raise prices.

There is a measure of the responsiveness of demand for product or service to a change in its price: elasticity of demand. This is defined as

\[
\text{elasticity of demand} = \frac{\text{percentage change in demand}}{\text{percentage change in price}}.
\]

To develop this formula, we’ll start by solving our demand function for \( x \), so that we have a function \( x = f(p) \). Then we have a demand function in terms of price. If we increase the price by \( h \) dollars, then the price is \( p + h \) and the quantity demanded is \( f(p + h) \).

The percentage change in demand is \( \frac{100}{f(p)} \left( \frac{f(p + h) - f(p)}{f(p)} \right) \) and the percentage change in price is \( \frac{100}{p} h \).

If we compute the ratio given above, we have \( \frac{100}{f(p)} \left( \frac{f(p + h) - f(p)}{f(p)} \right) \cdot \frac{100}{p} \).

We can simplify this to \( \frac{p}{f(p)} \left[ \frac{f(p + h) - f(p)}{h} \right] \).

For small values of \( h \), \( \frac{f(p + h) - f(p)}{h} \approx f'(p) \), so we have \( \frac{p \cdot f'(p)}{f(p)} \).

This quantity is almost always negative, and it would be much easier to work with a positive quantity. So the negative of this ratio is the elasticity of demand.

\[ E(p) = -\frac{p \cdot f'(p)}{f(p)}, \]

where \( p \) is price and \( f(p) \) is the demand function and is differentiable at \( x = p \).

Lesson 10 – Elasticity of Demand
Revenue responds to elasticity in the following manner:

If demand is **elastic** at $p$, then
- An increase in unit price will cause revenue to decrease or
- A decrease in unit price will cause revenue to increase

If demand is **unitary** at $p$, then
- An increase/decrease in unit price will cause the revenue to stay about the same.

If demand is **inelastic** at $p$, then
- An increase in the unit price will cause revenue to increase
- A decrease in unit price will cause revenue to decrease.

We have these generalizations about elasticity of demand:

Demand is said to be **elastic** if $E(p) > 1$.
Demand is said to be **unitary** if $E(p) = 1$.
Demand is said to be **inelastic** if $E(p) < 1$.

So, if demand is elastic, then the change in revenue and the change in price will move in opposite directions.

If demand is inelastic, then the change in revenue and the change in price will move in the same direction.

**Example 1:** Find $E(p)$ for the demand function $x + 2p - 15 = 0$ and determine if demand is elastic, inelastic or unitary when $p = 4$. 

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Lesson 10 – Elasticity of Demand
Example 2: Suppose the demand function for a product is given by \( p = -0.02x + 400 \). This function gives the unit price in dollars when \( x \) units are demanded.

a. Find the elasticity of demand.

b. Find \( E(100) \) and interpret the results.

c. Find \( E(300) \) and interpret the results.

d. If the unit price is $100, will raising the price result in an increase in revenues or a decrease in revenues?

e. If the unit price is $300, will raising the price result in an increase in revenues or a decrease in revenues?
What else can $E(p)$ tell you?

**Example 3:** If $E(p) = \frac{1}{2}$ when $p = 250$, what effect will a 1% increase in price have on revenue?

**Example 4:** If $E(p) = \frac{3}{2}$ when $p = 250$, what effect will a 1% increase in price have on revenue?