Now suppose the function, interval and/or subdivisions we wish to work with are not so nice. You would not want to work this type of problem by hand. We can work Riemann sum problems using GeoGebra. The command is:

\[
\text{RectangleSum[<Function>,<Start x-Value>,<End x-Value>,<Number of rectangles>,<Position for rectangle start>]}\]

“Position of rectangle start”: 0 corresponds to left endpoints, 0.5 corresponds to midpoints and 1 corresponds to right endpoints.

**Example 1:** Approximate the area between the x axis and the graph of 
\[
f(x) = 0.3x^3 - 0.807x^2 - 3.252x + 6.717\]
on \([-2.82, 1.33]\) with

*Enter the function in GGB.*

A. 50 rectangles and midpoints endpoints.

Command: Answer:

B. 10 rectangles and right endpoints.

Command: Answer:

**Example 2:** Approximate the area between the x axis and the function 
\[
f(x) = \frac{15}{0.083x^2 + 19.17x + 1}\]
on \([0.075, 8.21]\) using

*Enter the function in GGB.*

A. 12 rectangles and left endpoints.

Command: Answer:

B. 29 rectangles and midpoints.

Command: Answer:
Upper and Lower Sums Using GeoGebra

You can also find a related quantity using GeoGebra, the upper sum and/or the lower sum. Rather than always using the left endpoint, the right endpoint or the midpoint of the interval to find the height of the rectangle, the upper sum uses the biggest $y$ value on each interval as the height of the rectangle and the lower sum uses the smallest $y$ value on each interval as the height of the rectangle, no matter where on the interval that value occurs.

The command for Upper Sum is:
UpperSum[<Function>,<Start x-Value>,<End x-Value>,<Number of Rectangles>]

The command for Lower Sum is:
LowerSum[<Function>,<Start x-Value>,<End x-Value>,<Number of Rectangles>]

**Example 3:** Use GeoGebra to find the upper sum and the lower sum for $f(x) = \frac{1}{2}e^x + 2$ on the interval [-3, 5] using 35 rectangles. *Enter the function in GGB.*

Command: Answer:

Command: Answer:

The Definite Integral

Let $f$ be defined on $[a, b]$. If $\lim_{n \to \infty} \left( f(x_1) + f(x_2) + \ldots + f(x_n) \right) \Delta x$ exists for all choices of representative points in the $n$ subintervals of $[a, b]$ of equal width $\Delta x = \frac{b-a}{n}$, then this limit is called the definite integral of $f$ from $a$ to $b$. The definite integral is noted by $\int_a^b f(x) \, dx = \lim_{n \to \infty} \left( f(x_1) + f(x_2) + \ldots + f(x_n) \right) \Delta x$. The number $a$ is called the lower limit of integration and the number $b$ is called the upper limit of integration.