Popper 10 16. B 24. D

36. E

Exponential Decay $f(x) = a e^{-bx}$

Example 5: At the beginning of a study, there are 50 grams of a substance present. After 17 days, there are 38.7 grams remaining. Assume the substance decays exponentially.

a. State the two points given in the problem.

(0, 50) (17, 38.7)

Enter the two points in the spreadsheet and make a list.

b. Find an exponential regression model.

Command: Answer:

fitexp[list1] : $f(x) = 50e^{-0.0151x}$

c. What will be the rate of decay on day 40 of the study?

Command: Derivative Answer:

$f'(40)$ = -0.4126 grams/day

Exponential decay problems frequently involve the half-life of a substance. The **half-life** of a substance is the time it takes to reduce the amount of the substance by one-half.

Example 6: A certain drug has a half-life of 4 hours. Suppose you take a dose of 1000 milligrams of the drug. (time, substance)

a. State the two points given in the problem.

(0, 1000) (4, 500)

Enter the two points in the spreadsheet and make a list.

b. Find an exponential regression model.

Command: Answer:

fitexp[list1] $1000e^{-0.1733x}$

c. How much of it is left in your bloodstream 28 hours later?

Command: Answer:

$f(28)$ 7.1825 mg

Example 7: The half-life of Carbon 14 is 5770 years.

a. State the two points given in the problem.

(0, 100) (5770, 50)

think %

Enter the two points in the spreadsheet and make a list.

b. Find an exponential regression model.

Command: fitexp[list1]

Answer: $f(x) = 100e^{-0.0001x}$

c. Bones found from an archeological dig were found to have 22% of the amount of Carbon 14 that living bones have. Find the approximate age of the bones.

$f(x) = 22$ create $g(x) = 22$

Command: intersect[f, g]

Answer: $\boxed{x = 12{,}604}$

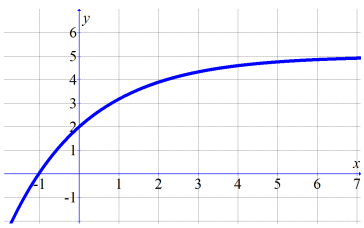
Limited Growth Models

Some exponential growth is limited; here's an example:

A worker on an assembly line performs the same task repeatedly throughout the workday. With experience, the worker will perform at or near an optimal level. However, when first learning to do the task, the worker's productivity will be much lower. During these early experiences, the worker's productivity will increase dramatically. Then, once the worker is thoroughly familiar with the task, there will be little change to his/her productivity.

The function that models this situation will have the form $Q(t) = C - Ae^{-kt}$.

This model is called a **learning curve** and the graph of the function will look something like this:



The graph will have a *y* intercept at $C - A$ and a horizontal asymptote at $y = C$. Because of the horizontal asymptote, we know that this function does not model uninhibited growth.

Example 8: Suppose your company's HR department determines that an employee will be able to assemble $Q(t) = 50 - 30e^{-0.5t}$ products per day, t months after the employee starts working on the assembly line. *Enter the function in GGB.*

a. How many units can a new employee assemble as s/he starts the first day at work?

$Q(0) = 20$

b. How many units should an employee be able to assemble after two months at work?

$Q(2) = 39$

c. How many units should an experienced worker be able to assemble?

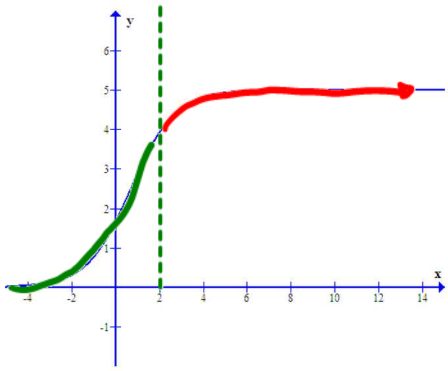
$\lim_{t \to \infty} Q(t) = 50$

d. At what rate is an employee's productivity changing 4 months after starting to work?

$Q'(4) = 2.03$

Logistic Functions

The last growth model that we will consider involves the logistic function. The general form of the equation is $Q(t) = \dfrac{A}{1 + Be^{-kt}}$ and the graph looks something like this:



If we looked at this graph up to around $x = 2$ and didn't consider the rest of it, we might think that the data modeled was exponential. Logistic functions typically reach a saturation point – a point at which the growth slows down and then eventually levels off. The part of this graph to the right of $x = 2$ looks more like our learning curve graph from the last example. Logistic functions have some of the features of both types of models.

Note that the graph has a limiting value at $y = 5$. In the context of a logistic function, this asymptote is called the **carrying capacity**. In general the carrying capacity is A from the formula above.

Logistic curves are used to model various types of phenomena and other physical situations such as population management. Suppose a number of animals are introduced into a protected game reserve, with the expectation that the population will grow. Various factors will work together to keep the population from growing exponentially (in an uninhibited manner). The natural resources (food, water, protection) may not exist to support a population that gets larger without bound. Often such populations grow according to a logistic model.

Example 9: A population study was commissioned to determine the growth rate of the fish population in a certain area of the Pacific Northwest. The function given below models the population where *t* is measured in years and *N* is measured in millions of tons. *Enter the function in GGB.*

$$N(t) = \frac{2.4}{(1+2.39e^{-0.338t})}$$

$$\frac{2.4}{1+2.39e^{-0.338}t}$$

a. What was the initial number of fish in the population?

N(0) = 0.708 million

b. What is the carrying capacity in this population?

"A" value = 2.4 million

c. What is the fish population after 3 years?

N(3) = 1.2855 million

d. How fast is the fish population changing after 2 years?

Derivative

N'(2) = 0.2009 million/yr

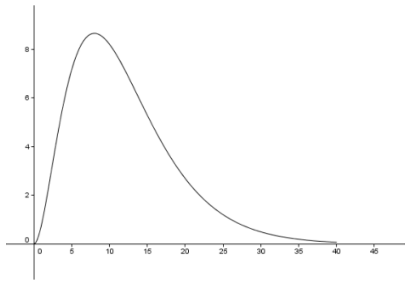
Hw 10 # 3 A & D

Open

Math 1314
Lesson 12
Curve Analysis (Polynomials)

This lesson will cover analyzing polynomial functions using GeoGebra.

Suppose your company embarked on a new marketing campaign and was able to track sales based on it. The graph below gives the number of sales in thousands shown *t* days after the campaign began.



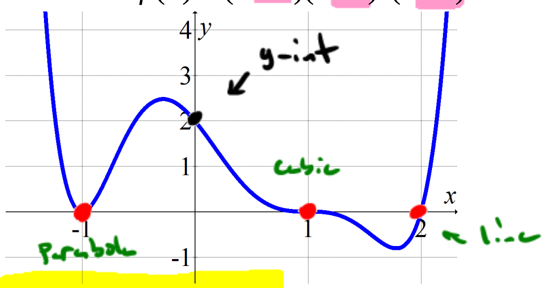
Now suppose you are assigned to analyze this information. We can use calculus to answer the following questions:

When are sales increasing or decreasing? (Note that the graph stops at $t = 40$.)
What is the maximum number of sales in the given time period?
Where does the growth rate change?
Etc.
Calculus can't answer the "why" questions, but it can give you some information you need to start that inquiry.

There will be several features of a polynomial function that we'll need to find. Let's start with a few College Algebra topics.

line cubic parabola

An example of a polynomial function is $f(x) = (x-2)(x-1)^3(x+1)^2$. Its graph looks like:



*The domain of any polynomial function is $(-\infty, \infty)$. Polynomial functions have no asymptotes.

*To find the roots/zeros/*x*-intercepts/solutions of any function, set the function equal to zero and solve for *x*. Or you may simply use the root or roots command in GGB.
*To find the *y*-intercept for any function, set $x = 0$ and calculate.
*The range of any polynomial is easier found by looking at the graph of the function.

x-int / roots / zero → where you cross the x-axis

Example 1: Let $f(x) = x^3 - 3x^2 - 13x + 15$. *Enter the function in GGB.*

a. Find any *x*-intercepts of the function.

Command: root[f]

Answer: (-3,0) (5,0) (1,0)

b. Find any *y*-intercept of the function. $x = 0$

Command: f(0)

Answer: = 15 ; (0,15)

Intervals on Which a Function is Increasing/Decreasing

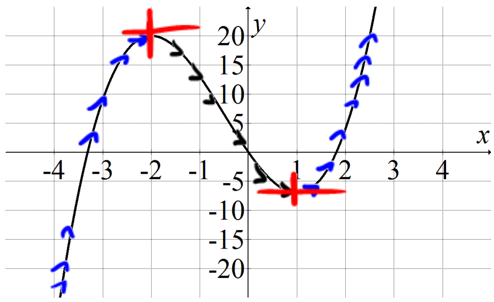
Definition: A function is **increasing** on an interval (*a*, *b*) if, for any two numbers x_1 and x_2 in (*a*, *b*), $f(x_1) < f(x_2)$, whenever $x_1 < x_2$. A function is **decreasing** on an interval (*a*, *b*) if, for any two numbers x_1 and x_2 in (*a*, *b*), $f(x_1) > f(x_2)$, whenever $x_1 < x_2$.

In other words, if the *y* values are getting bigger as we move from left to right across the graph of the function, the function is increasing. If they are getting smaller, then the function is decreasing. We will state intervals of increase/decrease using interval notation. The interval notation will consists of corresponding *x*-values wherever *y*-values are getting bigger/smaller.

Example 2: Given the following graph of a function, state the intervals on which the function is:

a. increasing. b. decreasing.

$(-\infty, -2) \cup (1, \infty)$



$(-2, 1)$

CN: -2, 1

We can use calculus to determine intervals of increase and intervals of decrease. A function can change from increasing to decreasing or from decreasing to increasing at its **critical numbers**, so we start with a definition of critical numbers:

The **critical numbers** of a polynomial function are all values of *x* that are in the domain of *f* where $f'(x) = 0$ (the tangent line to the curve is horizontal).

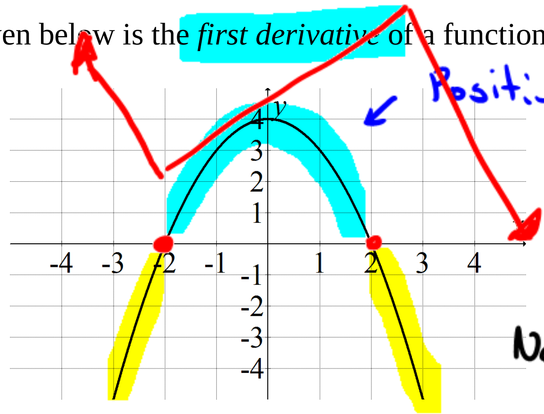
A function is **increasing** on an interval if the **first derivative** of the function is **positive** for every number in the interval. A function is **decreasing** on an interval if the **first derivative** of the function is **negative** for every number in the interval.

$-x^3$

$f'(x) = -3x^2$

Example 3: The graph given below is the *first derivative* of a function, *f*. Find:

$f'(x)$



Positive slopes in $f(x)$

Negative slopes in $f(x)$

a. any critical numbers. $f'(x) = 0$

$x = -2,\ 2$

$f(x)$

b. any intervals where the function is increasing/decreasing.

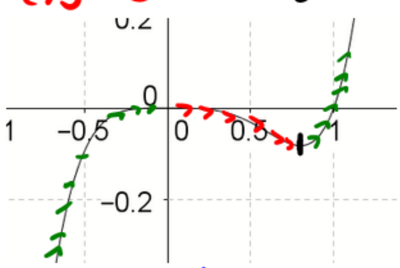
Increasing : $(-2, 2)$

Decreasing $(-\infty, -2) \cup (2, \infty)$

$f(x)$

Example 4: Let $f(x) = x^5 - x^4$, find: find $f'(x)$; $f'(x) = 0$

a. any critical numbers.



$f'(x) = 5x^4 - 4x^3$

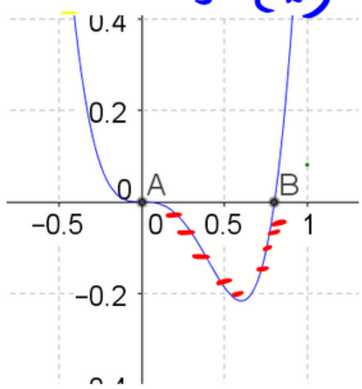
$(0, 0)$

$\text{root}\left[f'(x) \right]$

$\left(\frac{4}{5}, 0 \right)$

b. any intervals where the function is increasing/decreasing.

$f'(x)$



Increasing : $(-\infty, 0) \cup \left(\frac{4}{5}, \infty\right)$

Decreasing $\left(0, \frac{4}{5}\right)$

1. D

2. A