Rule 3: Derivative of a Constant Multiple of a Function

$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$ where c is any real number

Example 7: If $f(x) = -6x^4$, find $f'(x)$.

$f'(x) = -6 \cdot 4x^{4-1}$

$= -24x^3$

Example 8: If $f(x) = \frac{9}{5\sqrt[3]{x^{20}}}$, find $f'(x)$.

$f(x) = \frac{9}{5x^{20/3}} = \frac{9}{5} \cdot x^{-\frac{20}{3}}$

$f'(x) = \frac{\cancel{9}^{3}}{\cancel{5}_{1}} \cdot \frac{-\cancel{20}^{4}}{\cancel{3}_{1}} x^{-\frac{20}{3}-1}$ $\quad -\frac{20}{3} - 1 = -\frac{20}{3} - \frac{3}{3}$

$= -12x^{-\frac{23}{3}}$

$= \frac{-12}{x^{23/3}} = \boxed{\frac{-12}{\sqrt[3]{x^{23}}}}$

Question 41: If $f(x) = \sqrt[3]{x}$, find $f'(x)$

$\to x^{\frac{1}{3}}$

a. $\frac{1}{3}x^{\frac{2}{3}}$

b. $\frac{1}{x^{\frac{2}{3}}}$

c. $3x^{\frac{1}{3}}$

d. $\frac{1}{3x^{\frac{2}{3}}}$

$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1}$

$= \frac{1}{3}x^{-\frac{2}{3}}$

Rule 4: The Sum/Difference Rule

$$\frac{d}{dx}[f(x)\pm g(x)]=\frac{d}{dx}[f(x)]\pm\frac{d}{dx}[g(x)]$$

$-2x^{-6}$ $\quad$ $7x^{-1}$

Example 9: Find the derivative: $f(x)=-8x^4-\frac{2}{x^6}+\frac{7}{x}+10x-1.$

$$f'(x) = -8\cdot 4x^{4-1} - 2\cdot(-6)\,x^{-6-1} + 7(-1)\,x^{-1-1} + 10(1)\cdot x^{1-1} + 0$$

$$= -32x^3 + 12x^{-7} - 7x^{-2} + 10x^0$$

$$= -32x^3 + \frac{12}{x^7} - \frac{7}{x^2} + 10$$

Question 22:
If $f(x) = -\frac{2}{9}x^6$, find $f'(x)$.

$a. -\frac{12}{9}x^6$

$b. \ \frac{4}{3}x^5$

$c. -\frac{4}{3}x^5$

$d. -\frac{2}{9}x^5$

Let's revisit Example 1:

Example 10: Suppose the distance covered by a car can be measured by the function $s(t)=4t^2+32t$, where $s(t)$ is given in feet and t is measured in seconds. Find the instantaneous velocity of the car when $t=4$.

Recall that "instantaneous velocity" is the same as the derivative.

$$s'(t) = 4\cdot 2t^{2-1} + 32(1)t^{1-1}$$

$$s'(t) = 8t^1 + 32\cancel{t} \quad 1$$

$$s'(4) = 8(4) + 32 = 64 \text{ ft/s}$$

Note, there are many other rules for finding derivatives "by hand." We will not be using those in this course. Instead, we will use GeoGebra for finding more complicated derivatives.

$-2x^{-2}$

Question 34: Find the derivative:

$$f(x) = -2x^3 + 4x - \frac{2}{x^2}$$

a. $f'(x) = 6x^2 + 4 - \frac{4}{x^3}$

b. $f'(x) = -6x^2 + 4 + \frac{4}{x}$

c. $f'(x) = -6x^2 - 4 - \frac{2}{x}$

d. $f'(x) = -6x^2 + 4 + \frac{4}{x^3}$

$$f'(x) = -6x^2 + 4x^0 + 4x^{-3}$$
$$= -6x^2 + 4 + \frac{4}{x^3}$$

Math 1314
Lesson 7
Derivatives at a Point, Numerical Derivatives and Applications of the Derivative

Many of our problems will ask for the rate at which something is changing at a specific number. The number may express time, or quantity produced and sold, or many other quantities. To find this rate, find the derivative and substitute the desired number into the derivative and evaluate. State your answer using the correct units.

Example 1: A study conducted for a specific company showed that the number of lawn chairs assembled by the average worker t hours after starting work at 6 a.m. is given by
$N(t) = -t^3 + 7t^2 + 18t$. ← Variable needs to be x in GGB

a. Find the rate at which the average worker will be assembling lawn chairs t hr after starting work.

← Derivative

$$N'(t) = -3t^2 + 14t + 18$$

b. At what rate will the average worker be assembling lawn chairs at 9 a.m.?

← $t = 3$

$$N'(3) = -3(3)^2 + 14(3) + 18 = 33$$

Derivative of a Constant is zero

Example 2: The height of a rocket can be modeled by the function $h(t) = -16t^2 + 48t + 6$ where $h(t)$ gives the height in feet at time t given in seconds. At what rate is the height changing when $t = 1$?

← Derivative

$$h'(t) = -32t + 48$$

$$h'(1) = -32(1) + 48 = 16 \text{ ft/s}$$

We can find numerical derivatives using GGB.

Example 3: Find the numerical derivative of $f(x) = x^{\frac{3}{2}} - x$ when x = 4.
Enter the function into GGB.

Command: f`(4)

Answer: 2

Example 4: Find the numerical derivative of $C(x) = 0.000003x^3 - 0.04x^2 + 200x + 70000$ when x = 2500.

C(x)=0.000003x^3-0.04x^2+200x+70000

Enter the function into GGB.

Command: c`(2500)

Answer: 56.25

Example 5: Find the slope of the tangent line when x = -1 if $f(x) = \frac{x\sqrt{x+2}}{(2x+3)^2}$.

Derivative

(x*sqrt(x+2))/(2x+3)^2

Enter the function into GGB.

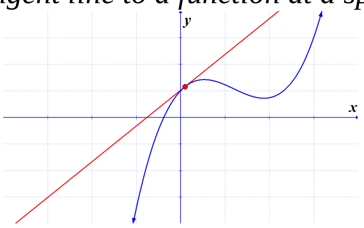
Command: g`(-1)

Answer: 4.5

Question 3: Find the numerical derivative of $f(x) = 2x^3 - 4x^2 + 8x - 10$ when $x = -1$

a. -24
b. -4
c. 6
d. 22
e. None of the above

We can also use the derivative to find the equation of a tangent line to a function at a specific point.
Recall the following graph of a tangent line to a function at a specific point.



Example 6: Let $f(x) = \frac{1}{3}x^3 - 2x^2 + 7x$. Write an equation of the tangent line to $f(x)$ at (3, 12)

$f'(x) = x^2 - 4x + 7$

$f'(3) = 4$ slope

"Rate of Change"

$y = mx + b$

$12 = (4)(3) + b$

$12 = 12 + b$

$0 = b$

$y = mx + b$

$y = 4x + 0$

Tangent line: $y = 4x$

Example 7: Let $f(x) = \frac{5}{3}x^3 + \frac{8}{3}$. Write an equation of the tangent line to $f(x)$ at $x = -1$.

12. C

For more complicated functions, we will use GGB.

Example 8: Write an equation of the line that is tangent to $f(x) = 1.6x^3 - 2.31x^2 + 6.39x - 2.81$ when $x = 3.75$.

Command: Answer:

In other cases, we may want to find all values of x for which the tangent line to the graph of f is horizontal. Since the slope of any horizontal line is 0, we'll want to find the derivative, set it equal to zero and solve the resulting equation for x.