A particle is moving on the perimeter of a circle with radius \( r = 10 \) with angular speed of \( \frac{\pi}{3} \) radians per second. After completing 3 full rotations, the particle traveled for 7 more seconds and stopped. What is the length of the total distance the particle traveled?

3 circles

1 circle = \( 2\pi r \)

= \( 2\pi \cdot 10 = 20\pi \)

3 rows + 3 \( \cdot 20\pi = 60\pi \)

\( \theta = \frac{\pi}{3} \cdot 7 = \frac{7\pi}{3} \) Central Angle

Sector/Arc length \( s = r\theta \)

= \( 10 \cdot \frac{7\pi}{3} = \frac{70\pi}{3} \)

\( 60\pi + \frac{70\pi}{3} \)

\( \frac{140\pi}{3} + \frac{70\pi}{3} = \frac{250\pi}{3} \)

\[ \sin \theta \left( \tan \theta + \cot \theta \right) \]

\[ = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \]

\[ = \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} = \frac{1}{\cos \theta} = \text{Sec } \theta \]
Section 5.1: Trigonometric Functions of Real Numbers

Opposite Angle Identities

The identities shown below are called **opposite-angle identities**. They tell us that the sine, tangent, cosecant and cotangent are **odd** functions and cosine and secant are **even** functions.

\[
\begin{align*}
\sin(-t) &= -\sin(t) \\
\csc(-t) &= -\csc(t) \\
\tan(-t) &= -\tan(t) \\
\cot(-t) &= -\cot(t) \\
\cos(-t) &= \cos(t) \\
\sec(-t) &= \sec(t)
\end{align*}
\]

**Example 1:** Use the opposite-angle identities to evaluate the following.

a. \( \cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \)

b. \( \tan\left(-\frac{\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) = -1 \)

**Example 2:** Simplify.  \( \cot(-t) \sec(-t) \)

\[
-\cot(t) \cdot \sec(t) = -\frac{\cos(t)}{\sin(t)} \cdot \frac{1}{\cos(t)} = -\frac{1}{\sin(t)} = -\csc(t)
\]

**Example 3:** Write an equivalent form: \( \csc(-6t) - \sec(-8t) \)

\( -\csc(6t) - \sec(8t) \)

**Example 4:** Suppose \( \sin(t) = -\frac{2}{3} \) and \( \pi < t < \frac{3\pi}{2} \). Find \( \tan(t) \) and \( \sec(t) \).

\[
\begin{align*}
\tan(t) &= \frac{-2}{-\sqrt{5}} = \frac{2\sqrt{5}}{5} \\
\sec(t) &= \frac{-3\sqrt{5}}{5}
\end{align*}
\]
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**Popper 48:** Select the appropriate quadrant \( \cot(\theta) < 0 \) and \( \csc(\theta) > 0 \)

a. Quadrant 1  
b. Quadrant 2  
c. Quadrant 3  
d. Quadrant 4

**Example 5:** Suppose \( \tan(t) = -\frac{\sqrt{3}}{2} \) and \( \frac{\pi}{2} < t < \pi \). Find \( \csc(t) \) and \( \cos(t) \).

```
csc(t) = \frac{\sqrt{3}}{1} = \frac{\sqrt{3}}{3}
```

```
cos(t) = \frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}
```

**Periodicity**

The circumference of the unit circle is \( 2\pi \). Thus, if we start with a point \( P \) on the unit circle and travel a distance of \( 2\pi \) units, we arrive back at the same point \( P \). That means that the arc lengths of \( t \) and \( t + 2\pi \) as measured from the point \( (1, 0) \) give the same terminal point on the unit circle. Thus, we have the following identities.

\[
\sin(t + 2k\pi) = \sin(t) \quad \text{Even } n
\]

\[
\cos(t + 2k\pi) = \cos(t)
\]

So,

\[
csc(t + 2k\pi) = \csc(t) \quad \text{Sec}(t + 2k\pi) = \sec(t)
\]

Like the sine and cosine functions, the tangent and cotangent functions also repeat themselves at intervals of lengths \( 2\pi \). In addition, the tangent and cotangent functions also repeat themselves at intervals of shorter length, namely \( \pi \). This, we have the following identities.

\[
\tan(t + k\pi) = \tan(t) \quad \text{Any } n
\]

\[
cot(t + k\pi) = \cot(t)
\]

For all real numbers \( t \) and all integers \( k \).

**Example 6:** Evaluate \( \sin\left(\frac{-20\pi}{3}\right) \).

```
Even \( \pi \)
```

\[
\frac{-20\pi}{3} = 6\pi - \frac{2\pi}{3}
\]

\[
\frac{-20\pi}{3} = -6\pi - \frac{2\pi}{3}
\]

\[
-\frac{20\pi}{3} = \frac{\sqrt{3}}{2}
\]
Example 7: Evaluate \( \cot \left( \frac{15\pi}{6} \right) \)

\[
\frac{15\pi}{6} = 2\pi + \frac{3\pi}{6}
\]

\[
\cot \left( \frac{3\pi}{6} \right) = \cot \left( \frac{\pi}{2} \right) = \frac{\cos \left( \frac{\pi}{2} \right)}{\sin \left( \frac{\pi}{2} \right)} = \frac{0}{1} = 0
\]

Example 8: Evaluate \( \frac{\cos \left( \frac{19\pi}{2} \right) \tan \left( \frac{21\pi}{4} \right)}{\cos(8\pi)} \)

\[
\frac{19\pi}{2} = 9\pi + \frac{\pi}{2}
\]

\[
\frac{21\pi}{4} = 5\pi + \frac{\pi}{4}
\]

\[
= \frac{\cos \left( 9\pi + \frac{\pi}{2} \right) \cdot \tan \left( \frac{5\pi}{4} \right)}{\cos(0)}
\]

\[
= \frac{\cos \left( 8\pi + \frac{3\pi}{2} \right) \cdot \tan \left( \frac{7\pi}{4} \right)}{\cos(0)}
\]

\[
= \frac{\cos \left( \frac{3\pi}{2} \right) \cdot \tan \left( \frac{\pi}{4} \right)}{\cos(0)}
\]

\[
= \frac{0 \cdot 1}{1} = 0
\]

Popper 36: Find the equivalent to the following. \( \frac{33\pi}{4} \)

a. \( 8\pi + \frac{\pi}{4} \)

b. \( 7\pi + \frac{\pi}{4} \)

c. \( 9\pi + \frac{3\pi}{4} \)

d. \( 8\pi + \frac{3\pi}{4} \)
Example 9: Evaluate \( \cot \left( \frac{15\pi}{4} \right) + \frac{\sin \left( \frac{10\pi}{3} \right)}{\cos \left( \frac{17\pi}{6} \right)} \)

\[
\frac{15\pi}{4} = 3\pi + \frac{3\pi}{4}
\]
\[
\frac{10\pi}{3} = 3\pi + \frac{\pi}{3}
\]
\[
\frac{17\pi}{6} = 2\pi + \frac{5\pi}{6}
\]

\[
\cot \left( 3\pi + \frac{3\pi}{4} \right) + \frac{\sin \left( 3\pi + \frac{4\pi}{3} \right)}{\cos \left( 2\pi + \frac{5\pi}{6} \right)}
\]
\[
= \cot \left( \frac{3\pi}{4} \right) + \frac{\sin \left( \frac{4\pi}{3} \right)}{\cos \left( \frac{5\pi}{6} \right)}
\]
\[
= -1 + \frac{-\frac{\sqrt{3}}{2}}{-\frac{\sqrt{3}}{2}}
\]
\[
= -1 + 1 = 0
\]
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Section 5.1b: Trigonometric Functions of Real Numbers

The following trigonometric identities are a direct consequence of the definition of the six trigonometric functions of the real number $t$:

1. $\tan(t) = \frac{\sin(t)}{\cos(t)}$
2. $\cot(t) = \frac{\cos(t)}{\sin(t)}$
3. $\csc(t) = \frac{1}{\sin(t)}$
4. $\sec(t) = \frac{1}{\cos(t)}$
5. $\cot(t) = \frac{1}{\tan(t)}$
6. $\sin^2(t) + \cos^2(t) = 1$

Identities (3) - (5) are known as the **reciprocal identities**. Identity (6) is known as the **Pythagorean identity**.

Two more Pythagorean identities are below. These identities can be obtained from identity (6).

7. $\tan^2(t) + 1 = \sec^2(t)$
   \[ \text{Divide both sides of (6) by } \cos^2(t) \]
8. $\cot^2(t) + 1 = \csc^2(t)$
   \[ \text{Divide both sides of (6) by } \sin^2(t) \]

To derive (7), begin by dividing both sides of (6) by $\cos^2(t)$ and divide by $\sin^2(t)$ to get (8).

**Example 1:** Simplify

a. $\frac{\tan(t)}{\sin(t)} = \frac{-\tan(t)}{\sin(t)}$

   

   $= \frac{-\sin(t)}{\cos(t)} \cdot \frac{1}{\sin(t)}$

   

   $= -\frac{1}{\cos(t)} = -\sec(t)$
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b. \( \cot(t) - \csc(t) \)

c. \( \sin(t) \cot(t) + \cos(t) \)

d. \( \sin(t) \cot(t) - \cos(t) \)

Recall:
- \( \sin(t + 2\pi k) = \sin(t) \)
- \( \cos(t + 2\pi k) = \cos(t) \)
- \( \csc(t + 2\pi k) = \csc(t) \)
- \( \sec(t + 2\pi k) = \sec(t) \)
- \( \tan(t + 2\pi k) = \tan(t) \)
- \( \cot(t + 2\pi k) = \cot(t) \)

For all real numbers \( t \) and all integers \( k \).

Example 2: Simplify.

a. \( \cos(-t) \cot(-t) + \sin(-t) \)

b. \( \sin(t) \cot(t) + \cos(t) \)

c. \( \cot(t) - \csc(t) \)

d. \( \sin(t) \cot(t) - \cos(t) \)