Example 1: Evaluate the following.

\[
\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \quad \tan\left(-\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{3} \quad \cot\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{3}
\]

\[
\cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2} \quad \sec\left(\frac{5\pi}{6}\right) = \frac{2\sqrt{3}}{3} \quad \csc\left(\frac{\pi}{3}\right) = \frac{2\sqrt{3}}{3}
\]

Example 2: Simplify / Evaluate

a. \[\sin\left(\frac{14\pi}{3}\right) = \sin\left(\frac{4\pi + \frac{2\pi}{3}}{3}\right) = \sin\left(\frac{2\pi}{3}\right)\]

\[\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}\]

b. \[\tan\left(-\frac{16\pi}{3}\right) = \tan\left(-\frac{5\pi}{3} - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}\]

\[\frac{-\sqrt{3}}{3} = \frac{-\sqrt{3}}{3}\text{ with the same sign}\]

c. \[4\cos\left(\frac{34\pi}{6}\right) + \cot\left(\frac{17\pi}{4}\right)\]

\[4\cos\left(\frac{5\pi}{3} + \frac{4\pi}{3}\right) + \cot\left(\frac{4\pi}{3} + \frac{\pi}{4}\right)\]

\[4\cos\left(\frac{\pi}{3} + \frac{19\pi}{3}\right) + \cot\left(\frac{3\pi}{4}\right)\]

\[4\cos\left(\frac{5\pi}{3}\right) + 1\]

\[4 \cdot \left(\frac{1}{2}\right) + 1 = 3\]
Example 3: Evaluate

\[
\sin^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{3}
\]

\[
\tan^{-1}(-1) = -\frac{\pi}{4}
\]

\[
\cot^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}
\]

\[
\sec^{-1}\left(\frac{2\sqrt{3}}{3}\right) = \frac{\pi}{2}
\]

\[
\csc^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}
\]

\[
\cos^{-1}\left(-\frac{2}{3}\right)
\]

\[
\cos^{-1}\left(\frac{3}{2}\right)
\]

\[
[0,\pi]
\]

\[
[0,\frac{\pi}{2})\cup(\frac{\pi}{2},\pi]
\]

Example 4: Evaluate

\[
\cot\left[\cos^{-1}\left(-\frac{3}{7}\right)\right]
\]

\[
\cos^{-1}\left(\frac{4}{5}\right) = [0,\frac{\pi}{2})
\]

\[
\cot\left(\frac{4}{5}\right) = \frac{\text{adj}}{\text{opp}} = \frac{-3}{2\sqrt{5}} = -\frac{3\sqrt{5}}{10}
\]

Example 5: Find the amplitude, period and phase shift.

\[
4\cos\left(\frac{1}{3}x\right)
\]

Amplitude: \(4\)

Period: \(\frac{2\pi}{\frac{1}{3}} = \frac{6\pi}{1} = 6\pi\)

Phase Shift: None

\[
-\frac{1}{2}\sin\left(\frac{2\pi}{3}x + \frac{3\pi}{2}\right)
\]

Amplitude: \(\frac{1}{2}\)

Period: \(\frac{2\pi}{\frac{2\pi}{3}} = \frac{3}{1} = 3\)

Phase Shift: \(\frac{\pi}{2} = \frac{\frac{3\pi}{2}}{2\pi/3} = \frac{9}{4} - \frac{9}{4}\) to the left
Example 6: List all the x-intercepts for

\[ y = 4 \cos \left( 4x + \frac{\pi}{3} \right) \]
on the \([-\pi/6, \pi/2]\)

\[ \cos \theta = 0 \]
\[ \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \]

\[ 4x + \frac{\pi}{3} = \frac{\pi}{2} \]
\[ 4x = \frac{\pi}{6} \]
\[ x = \frac{\pi}{24} \]

\[ 4x + \frac{\pi}{3} = \frac{3\pi}{2} \]
\[ 4x = \frac{7\pi}{6} \]
\[ x = \frac{7\pi}{24} \]

\[ 4x + \frac{\pi}{3} = \frac{5\pi}{2} \]
\[ 4x = \frac{11\pi}{6} \]
\[ x = \frac{11\pi}{24} \]

Example 7: Write a cosine function with a positive vertical dilation, given the amplitude is 3, the phase shift is 3 to the left, the vertical shift is 2 up, and the period is 2.
5. Given the right triangle ABC and ADC with right angle C, \( m(\overline{BAD}) = \alpha \) and \( m(\overline{ADC}) = \beta \).

If \( \sin \alpha = \frac{1}{5} \) and \( \sin \beta = \frac{1}{4} \);

Find \( \tan \left( \overline{ABC} \right) \). (You do not need to simplify your answer.)

\[
\tan (\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}
\]

\[
\begin{align*}
&= \frac{\frac{1}{250} + \frac{1}{\sqrt{15}}}{1 - \frac{1}{250} \cdot \frac{1}{\sqrt{15}}} \\
&= \frac{\frac{\sqrt{15} + 250}{250 \sqrt{15}}}{\frac{250 - \sqrt{15}}{250 \sqrt{15}}} \\
&= \frac{\sqrt{15} + 250}{250 - 1}
\end{align*}
\]
Example 8: Sketch the following functions: Label, for one period, x-, y-intercepts as ordered pairs; max value(s), min value(s) as ordered pairs.

\[ f(x) = -3 \sin(4x) \]

\[ g(x) = 7 \cos(5x) \]
Example 9: Given \( \sin x = \frac{2}{5} \), with \( 90^\circ < x < 180^\circ \), and \( \sin y = -\frac{1}{3} \) with \( 180^\circ < y < 270^\circ \). Find:

\[
\sin(x + y)
\]

\[
\sin(x - y)
\]

\[
\cos(x + y)
\]

\[
\cos(x - y)
\]

\[
\sin(2x)
\]
Review Test 3

\[ \cos(2x) \]

**Example 10:** Given \( \cos x = \frac{1}{4} \) with \( 270^\circ < x < 360^\circ \). Find \( \tan(2x) \)

**Example 11:** Suppose that \( \sec(x) = \frac{8}{7} \) and that \( 0^\circ < x < 90^\circ \). Compute \( \sin(-x) \).
Example 12: Determine all solutions to $\sin 3\theta = \frac{1}{2}$ on the interval $[0, 2\pi)$.

Example 13: Solve the following equation on the interval $[0, 2\pi)$.

$$3 \cos^2 x - 8 \cos x + 5 = 0$$

Example 14: Solve the following equation on the interval $[0, 2\pi)$.

$$2 \sin^2 x + \sin x - 1 = 0$$
Review Test 3

**Example 15:** Find using the sum or difference formulas.

a. \( \cos(15^\circ) \)

b. \( \sin(75^\circ) \)

**Example 16:** Given \( \sin(x) = \frac{\sqrt{11}}{6} \) where \( x \) is an acute angle. Find \( \sin\left(\frac{x}{2}\right) \).