Beginning of the Semester
To-Do List

- Set up your account at https://casa.uh.edu/

- Read the Math 13xx Departmental Course Policies

- Take “Course Policies Quiz” until your score is 100%. You can find it on the “Online Assignments” tab at https://casa.uh.edu/ after you set up your account.

- Take Practice Test 1 (twenty times) until you master it by the deadline, and schedule a time to take Test 1 at CASA testing centers. The scheduling process can be completed through “Proctored Exams” tab on your CASA account.

- Learn your Grading ID Number (Peoplesoft ID number).

- Find the online textbook link at https://casa.uh.edu/

- Buy an Access Code at the register at the UH Bookstore. Log on to https://casa.uh.edu/ and follow the steps to enter this Access Code. Deadline to enter the code: Sunday, September 4th, 2016.

- Buy a package of bubbling forms at the UH Bookstore. These must have your course’s section number printed on them. You will need these forms beginning the 3rd week of classes. (Online Sections do not need this package of bubbling forms.)
Most of this course will deal with functions. Suppose we start with two sets, $A$ and $B$. A function is a rule which assigns one and only one element of set $B$ to each element in set $A$. Set $A$ is called the **domain** of the function and set $B$ is called the **range**.

We’ll start by looking at **mappings**.

A mapping relates each element in the oval on the left with an element in the oval on the right. You need to be able to state whether or not the mapping defines a function, and, if it defines a function, you should be able to state the domain and range of the function.

**Example 1:** State whether or not the mapping represents a function. If it does, identify its domain and range.

![Mappings Diagram]

Functions are usually written using function notation. If an equation is solved for $y$, such as $y = mx + b$, we would write this using function notation as $f(x) = mx + b$, read “$f$ of $x$,” denoting the value of the function at $x$. We can also use other lower case letters to denote a function, such as $g, h, j, k$, etc.

Often, you will be asked to state the domain of a stated function. Domain is a subset of the set of real numbers.
Math 1330 Algebra Review

Reminder: Interval Notation

(-3, 5) all $x$ such that $-3 < x < 5$

[-3, 5] all $x$ such that $-3 \leq x \leq 5$

[-3, 5) all $x$ such that $-3 \leq x < 5$

[-3, $\infty$) all $x$ such that $x \geq -3$

(-$\infty$, 5) all $x$ such that $x < 5$

(-$\infty$, $\infty$) all real numbers

Next, we’ll state the domain of several types of functions.

The domain of any **polynomial function** is $(-\infty, \infty)$, or all real numbers.

The domain of any **rational function**, where both the numerator and the denominator are polynomials, is all real numbers except the values of $x$ for which the denominator equals 0.

**Example 2:** State the domain of the function. Write your answer using interval notation.

$$f(x) = \frac{x - 3}{x + 7}$$

$$x + 7 = 0$$
$$x = -7$$

Exclude this $x = -7$

D: $(-\infty, -7) \cup (-7, \infty)$

The domain of any radical function with **even index** is the set of real numbers for which the radicand is greater than or equal to 0. The domain of any radical function with **odd index** is $(-\infty, \infty)$.

**Example 3:** State the domain of the function. Write your answer using interval notation.

$$h(x) = \sqrt{x + 4}$$

$$x + 4 \geq 0$$
$$x \geq -4$$

D: $[-4, \infty)$
Example 4: State the domain of the function. Write your answer using interval notation.

\[ g(x) = \frac{x - 1}{x - 5} \]

\[ x - 5 \neq 0 \quad x - 1 \geq 0 \quad x \geq 1 \]

\[ D: [1, 5) \cup (5, \infty) \]

Increasing/Decreasing

A function is increasing on an interval \((a, b)\) if \(f(x_1) < f(x_2)\) for each \(x_1 < x_2\) in \((a, b)\). You can think: a function is increasing if the \(y\) values are getting bigger as we look from left to right.

A function is decreasing on an interval \((a, b)\) if \(f(x_1) > f(x_2)\) for each \(x_1 > x_2\) in \((a, b)\). You can think: a function is decreasing if the \(y\) values are getting smaller as we look from left to right.

A turning point is a point where the graph of a function changes from increasing to decreasing or where it changes from decreasing to increasing. Peak or Valley

Example 5: Identify any turning points, and intervals of increasing and decreasing on the graph of this function.

Inc: \((-\infty, -2) \cup (3, \infty)\)
Dec: \((-2, 3)\)
Math 1330 Algebra Review

Even and Odd Functions

A function \( f \) is **even** if \( f(-x) = f(x) \) for all \( x \) in the domain of \( f \). Since an even function is symmetric with respect to the \( y \)-axis, the points (\(-x, y\)) and (\(x, y\)) are on the same graph.

\[
f(x) = x^2
\]

A function is **odd** if \( f(-x) = -f(x) \) for all \( x \) in the domain of the function. Odd functions have symmetry with respect to the origin. Here is an example of an odd function.

**Example 6:** Determine if \( f(x) = 5x^4 - 3x^2 + 2 \), is odd, even or neither.

\[
f(-x) = f(x)
\]

\[
f(-x) = 5(-x)^4 - 3(-x)^2 + 2
\]

\[
= 5x^4 - 3x^2 + 2 \quad \text{(Original Function)}
\]

**Even Function**
Example 7: Determine if \( f(x) = x^3 - x + 1 \), is odd, even or neither.

\[
\begin{align*}
\text{Test for oddness: } & \quad f(-x) = -f(x) \\
& \quad f(-x) = (-x)^3 - (-x) + 1 \\
& \quad = -x^3 + x + 1 \quad \text{This is } -1 \cdot f(x)
\end{align*}
\]

\[ -1 \cdot [x^3 - x + 1] \]

\[ -x^3 + x - 1 \]

Neither

Operations on Functions

Let \( f \) and \( g \) be two functions. The following are functions whose domains are the set of real numbers common to the domain of \( f \) and \( g \), defined as follows:

**Sum:** \( (f + g)(x) = f(x) + g(x) \)

**Difference:** \( (f - g)(x) = f(x) - g(x) \)

**Product:** \( (fg)(x) = f(x)g(x) \)

**Quotient:** \( \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0 \)

**Composition:** \( (f \circ g)(x) = f(g(x)) \)

The domain of the composition \( f \circ g \) is the set of all \( x \) such that

1. \( x \) is in the domain of \( g \) (the "inside" function)
2. \( g(x) \) is in the domain of \( f \) (the "outside" function)

Example 8: Suppose \( f(x) = x^2 - 4x + 3 \) and \( g(x) = 3x - 1 \).

a. \( f(g(2)) \)

\[
\begin{align*}
\text{Let } g(2) &= 3(2) - 1 \\
&= 5 \\
\text{Then } f(g(2)) &= f(5) \\
&= (5)^2 - 4(5) + 3 \\
&= 25 - 20 + 3 \\
&= 8
\end{align*}
\]
Math 1330 Algebra Review

b. \( g(f(x)) = g(x^2 - 4x + 3) = 3 \left( x^2 - 4x + 3 \right) - 1 \)

\[
= 3x^2 - 12x + 9 - 1 \\
= 3x^2 - 12x + 8
\]

Example 9: The graphs of two functions, \( f \) and \( g \), are shown below. Find the following:

\[ (f \circ g)(2) = f(g(-1)) = f(-1) \]

\[ g(-2) = -1 \]

\[ f(-1) = 2 \]

b. \( g(f(-3)) = (g \circ f)(-3) = g(0) = -5 \)

\[ f(-3) = 0 \]
Next, you will need to be able to form a difference quotient. You will need to compute the following in a few steps.

\[
\frac{f(x + h) - f(x)}{h}, \quad h \neq 0
\]

**Example 10:** Find the difference quotient: \( f(x) = x^2 - 2x - 9 \)

\[
\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) - 9 - (x^2 - 2x - 9)}{h}
\]

\[
= \frac{2xh + h^2 - 2h}{h}
\]

\[
= 2x + h - 2
\]

**Example 11:** Find the difference quotient:

\[
f(x) = \frac{2x + 3}{x - 4}
\]

\[
\frac{f(x+h) - f(x)}{h} = \frac{\frac{2(x+h) + 3}{(x+h - 4)} - \frac{2x + 3}{(x-4)}}{h}
\]

\[
= \frac{(x-4)(2x+2h+3) - (x+h-4)(2x+3)}{h \cdot (x+h-4)(x-4)}
\]

\[
= \frac{2x^2 + 2xh + 3x - 8x - 12 - \left(2x^2 + 3x + 2hx + 3h - 8x - 12\right)}{h \cdot (x+h-4)(x-4)}
\]

\[
= \frac{-2h}{h \cdot (x+h-4)(x-4)} = \frac{-2}{(x+h-4)(x-4)}
\]

\[
= \frac{-1}{(x+h-4)(x-4)}
\]
Inverse Function

The inverse function of a one-to-one function is a function \( f^{-1}(x) \) such that \( (f \circ f^{-1})(x) = x \).

To determine if two functions are inverses of one another, you need to compose the functions in both orders. Your result should be \( x \) in both cases. That is, given two functions \( f \) and \( g \), the functions are inverses of one another if and only if \( f(g(x)) = g(f(x)) = x \).

Note that if \( A \) is the domain of \( f \) and \( B \) is the range of \( f \), then the domain of \( f^{-1} \) is \( B \) and the range of \( f^{-1} \) is \( A \).

You need to be able to find the inverse of a function. Follow this procedure to find an inverse function:

1. Rewrite the function as \( y = f(x) \).
2. Interchange \( x \) and \( y \).
3. Solve the equation you wrote in step 2 for \( y \).
4. Rewrite the inverse using inverse notation, \( f^{-1}(x) \).

Example 12: Find the inverse of the following function.

\[
 f(x) = \frac{3 + x}{2 - x}
\]

\[
 y = \frac{3 + x}{2 - x}
\]

\[
 x = \frac{3 + y}{2 - y}
\]

\[
 x (2-y) = 3 + y
\]

\[
 2x - xy = 3 + y
\]

\[
 -y - xy = 3 - 2x
\]

\[
 y(1-x) = 3 - 2x
\]

\[
 y = \frac{3 - 2x}{-1 - x}
\]

\[
 f^{-1}(x) = \frac{3 - 2x}{-1 - x}
\]

\[
 0c\]

\[
 \frac{3 - 2x}{-1(1+x)} = -\frac{3 - 2x}{1 + x}
\]

\[
 0c\]

\[
 = \frac{2x - 3}{1 + x}
\]
Example 13: Find the inverse of the following function.

\[ f(x) = \frac{3}{\sqrt{x} + 4} \]

\[
\begin{align*}
y &= \frac{3}{\sqrt{x} + 4} \\
x &= \frac{3}{\sqrt{y} + 4} \\
(x^3)^3 &= (\frac{3}{\sqrt{y} + 4})^3 \\
x^3 &= y + 4 \\
x^3 - 4 &= y
\end{align*}
\]

\[ f^{-1}(x) = x^3 - 4 \]

Example 14: If \( f(-1) = 2, f^{-1}(-1) = 0 \) and \( f(2) = 5 \), find \( f(0) \) and \( f^{-1}(5) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f^{-1}(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>11</td>
<td>5</td>
</tr>
</tbody>
</table>

\[ f(0) = -1 \]
\[ f^{-1}(5) = 2 \]

Polynomial Function

A polynomial function is a function which can be written in the form

\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 \]

The numbers \( a_0, a_1, \ldots, a_n \) are called the coefficients of the polynomial function and \( a_n \neq 0 \).

\( x^n \) is the largest variable term of the polynomial function of \( x \) of degree \( n \). The number \( a_n \), is the coefficient of the variable to the highest power, is called the leading coefficient.

Note: The variable is only raised to positive integer powers—no negative or fractional exponents. However, the coefficients may be any real numbers, including fractions or irrational numbers like \( \pi \). The domain of any polynomial function is all real numbers.

Example 15: Given \( f(x) = -3x^6 - 5x^4 + 4x^3 - 2x^2 - 9x + 11 \). What is the degree and leading coefficient?

Degree: 6
LC: -3
Facts about polynomials:
- They are smooth curves, with no jumps or sharp points.
- A polynomial has at most \( n - 1 \) turning points.
- A polynomial has at most \( n \) \( x \)-intercepts.
- A polynomial has exactly one \( y \)-intercept.

**End Behavior of Polynomial Functions**

The behavior of a graph of a function to the far left or far right is called its **end behavior**.

**Even Degree**

- Positive Leading Coefficient
- Negative Leading Coefficient

**Odd Degree**

- Positive Leading Coefficient
- Negative Leading Coefficient

**Zeros of polynomials**

If \( f \) is a polynomial and \( c \) is a real number for which \( f(c) = 0 \), then \( c \) is called a **zero** of \( f \), or a **root** of \( f \).

If \( c \) is a zero of \( f \), then
- \( c \) is an \( x \)-intercept of the graph of \( f \)
- \( (x - c) \) is a factor of \( f \).
So if we have a polynomial in factored form, we know all of its x-intercepts.

- every factor gives us an x-intercept.
- every x-intercept gives us a factor.

Note: In factoring the equation for the polynomial function \( f \), if the same factor \( x - c \) occurs \( k \) times, we call \( c \) a repeated zero with multiplicity \( k \).

**Description of the Behavior at Each x-intercept**

1. Even Multiplicity: The graph touches the x-axis, but does not cross it (looks like a parabola there).
2. Odd Multiplicity of 1: The graph crosses the x-axis (looks like a line there).
3. Odd Multiplicity greater than or equal to 3: The graph crosses the x-axis and it looks like a cubic there.

**Rational Functions**

Definition: A **rational function** is a function that can be written in the form \( f(x) = \frac{P(x)}{Q(x)} \), where \( P(x) \) and \( Q(x) \) are polynomials, consists of all real numbers \( x \) such that \( Q(x) \neq 0 \).

You will need to be able to find the following:

- Domain
- Intercepts
- Holes
- Vertical asymptotes
- Horizontal asymptotes
- Slant asymptotes
- Behavior near the vertical asymptotes

**Domain:** The domain of \( f \) is all real numbers except those values for which \( Q(x) = 0 \).

**x-intercept(s):** All values of \( x \) for which \( P(x) = 0 \).

**y-intercept:** The \( y \) intercept of the function is \( f(0) \).

**Holes:** The graph of the function will have a hole if there is a common factor in the numerator and denominator.

**Vertical asymptotes:** The graph of the function has a vertical asymptote at any value of \( x \) for which \( Q(x) = 0 \) and \( P(x) \neq 0 \).
Example 16: Given the following function. Find the domain, x and y intercepts, hole(s) if any and vertical asymptotes if any.

\[
f(x) = \frac{x^2 + x - 6}{x^3 + 2x^2 - 3x}
\]

\[
= \frac{(x+3)(x-2)}{x(x-3)(x+1)} = \frac{x-2}{x(x-1)}
\]

**Hole:** \(x+3 = 0\)
\(x = -3\)

**Domain:** \(x \neq -3, 0, 1\)

**x-intercept:** Numerator = 0
\(x - 2 = 0\)
\(x = 2\)

**y-intercept:** \(f(0) = \text{undefined}\)

**Vertical Asymptotes:**
- Denominator = 0
  - \(x = 0\)
  - \(x - 1 = 0\)
  - \(x = 1\)

**Horizontal asymptotes:** You can determine if a graph of a function has a horizontal asymptote by comparing the degree of the numerator with the degree of the denominator.

- Shorthand: degree of \(f = \text{deg}(f)\), numerator = \(N\), denominator = \(D\)
  1. If \(\text{deg}(N) > \text{deg}(D)\) then there is no horizontal asymptote.
  2. If \(\text{deg}(N) < \text{deg}(D)\) then there is a horizontal asymptote and it is \(y = 0\) (x-axis).
  3. If \(\text{deg}(N) = \text{deg}(D)\) then there is a horizontal asymptote and it is \(y = \frac{a}{b}\), where \(a\) is the leading coefficient of the numerator and \(b\) is the leading coefficient of the denominator.

**Slant asymptote:** The graph of the function may have a slant asymptote if the degree of the numerator is one more than the degree of the denominator. To find the equation of the slant asymptote, use long division to divide the denominator into the numerator. The quotient is the equation of the slant asymptote.

**Behavior near the vertical asymptotes:** The graph of the function will approach either \(\infty\) or \(-\infty\) on each side of the vertical asymptotes. To determine if the function values are positive or negative in each region, find the sign of a test value close to each side of the vertical asymptotes.
Note: It is possible for the graph to cross the horizontal asymptote, maybe even more than once. To figure out whether it crosses (and where), set y equal to the y-value of the horizontal asymptote and then solve for x.

**Example 17:** Does the function cross the horizontal asymptote?

a. 
\[ f(x) = \frac{2x^2 - 2x - 3}{3x^2 + x - 6} \]

b. 
\[ f(x) = \frac{1 - 2x}{x + 1} \]
More Review...

Exponential Functions

Functions whose equations contain a variable in the exponent are called **exponential functions**.

The function \( f(x) = a^x \) is the exponential function with base \( a > 0 \) and \( a \neq 1 \).

We’ll look at two cases of the exponential function, \( a > 1 \) and \( 0 < a < 1 \).

For \( a > 1 \):

- **Domain**: \((-\infty, \infty)\)
- **Range**: \((0, \infty)\)
- **Key point**: \((0, 1)\)
- **Horizontal asymptote**: \( y = 0 \) since \( y \to 0 \) as \( x \to -\infty \)

The graph of \( f(x) = a^x \) with \( a > 1 \) has this shape (larger \( a \) results in a steeper graph):

![Graph of \( f(x) = a^x \) with \( a > 1 \)].

The graph of \( f(x) = a^x \) with \( 0 < a < 1 \) has this shape (smaller \( a \) results in a steeper graph):

![Graph of \( f(x) = a^x \) with \( 0 < a < 1 \)].
The Number “e”

More on transformations of the exponential function \( f(x) = a^x \), but with \( a = e \) (the natural base).

**Definition:** \( e \) is the “limiting value” of \( \left(1 + \frac{1}{x}\right)^x \) as \( x \) grows to infinity.

\( e \approx 2.718281282459 \). It is an irrational number, like \( \pi \). This means it cannot be written as a fraction nor as a terminating or repeating decimal.

**Example 1:** Solve for \( x \).

a.) \( 5^{3x-1} = 125 \)

b.) \( 8^{5x-2} = 32 \)
Logarithmic Functions

The exponential function is 1-1; therefore, it has an inverse function. The inverse function of the exponential function with base \(a\) is called the **logarithmic function with base** \(a\).

For \(x > 0\) and \(a > 0\) and \(a\) not equal to 1, \(y = \log_{a} x\) is equivalent \(a^{y} = x\)

The function \(f(x) = \log_{a} x\) is the **logarithmic function with base** \(a\)

The **common logarithm** is the logarithm with base 10. We denote this as \(\log_{10} x = \log x\)

The **natural logarithm** is the logarithm with base \(e\). We denote this as \(\log_{e} x = \ln x\)

**Laws of Logarithms**

If \(m, n\) and \(a\) are positive numbers, \(a \neq 1\), then

1. \(\log_{a} mn = \log_{a} m + \log_{a} n\)
2. \(\log_{a} \frac{m}{n} = \log_{a} m - \log_{a} n\)
3. \(\log_{a} m^{n} = n \log_{a} m\)
4. \(\log_{a} 1 = 0\)
5. \(\log_{a} a = 1\)
6. \(\log_{a} a^{x} = x\)
7. \(a^{\log_{a} x} = x\)
8. \(\log_{a} m = \frac{\log m}{\log a}\) (change of bases formula)

**Characteristics of the Graphs of Logarithmic Functions of the Form** \(f(x) = \log_{a} x\)

1. The \(x\)-intercept is \((1, 0)\) and there is no \(y\)-intercept.
2. The \(y\)-axis is a vertical asymptote.
3. The domain is all positive real numbers.
4. The range is all real numbers.

If \(a > 1\), the graph of \(f(x) = \log_{a} x\) looks like:
If $0 < a < 1$, the graph of $f(x) = \log_a x$ looks like:

Note: If a logarithmic function is translated to the left or to the right, the vertical asymptote is shifted by the amount of the horizontal shift.

**Example 2:** Evaluate, if possible.

\[
\begin{align*}
\log_6 36 & \quad \log_2 \frac{1}{8} & \quad \log_5 125 \\
\log 100 & \quad \log_4 2 & \quad \log 0.001 \\
\log_3 (\sqrt[3]{81}) & \quad \log_5 \sqrt{125} \\
\end{align*}
\]

**Example 3:** Solve for $x$: $5^{3x} = 9$. (a) Give the exact value using natural logarithms.
Example 4: Solve for $x$: $4e^{(x+5)} + 5 = 7$. (a) Give the exact value using natural logarithms.

Example 5: Solve for $x$: $e^{2x} - 9e^x + 20 = 0$. (a) Give the exact value using natural logarithms.
Example 6: Solve for $x$: $25^{3x-2} = \frac{1}{(\sqrt{125})^x}$

(a) Give the exact value using natural logarithms.

Example 7: Solve for $x$: $\log_6(x) + \log_6(x + 1) = \log_6 2$

Example 7: Solve for $x$: $\log_6 x + \log_6(5 - x) = 2$