Evaluating Trigonometric Functions Using Reference Angles

1. Determine the reference angle associated with the given angle.
2. Evaluate the given trigonometric function of the reference angle.
3. Affix the appropriate sign determined by the quadrant of the terminal side of the angle in standard position.

**Example 6:** Evaluate the following.

a. \( \sin(300^\circ) \)
   
   Reference Angle: \( 60^\circ \)
   
   \( \sin(300^\circ) = -\sin(60^\circ) \)
   
   \[ \frac{-\sqrt{3}}{2} \]

b. \( \cos(-240^\circ) \)
   
   Reference Angle: \( 60^\circ \)
   
   \( \cos(-240^\circ) = -\cos(60^\circ) \)
   
   \[ \frac{1}{2} \]
c. \( \sec(135^\circ) \)  

In Q2, \( \sec \theta \) is negative.

RA: 45° \( \sec(135^\circ) = -\sec(45^\circ) = \frac{-\sqrt{2}}{\sqrt{2}} = -1 \)

Question 2: \( \cos\left(\frac{3\pi}{4}\right) \)

a. \( \frac{\sqrt{2}}{2} \)

b. \( -\frac{\sqrt{2}}{2} \)

c. \( \frac{\sqrt{3}}{2} \)

d. \( -\frac{\sqrt{3}}{2} \)

e. \( \sin\left(\frac{4\pi}{3}\right) \)

in Q3, \( \sin \theta \) is negative.

\( \sin\left(\frac{4\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \)

f. \( \tan(90^\circ) = \text{undefined} \)

\( \frac{\sin(90^\circ)}{\cos(90^\circ)} = \frac{1}{0} = \text{undefined} \)

g. \( \cot(-7\pi) = \cot(-\pi) = \frac{\cos(-\pi)}{\sin(-\pi)} = \frac{-1}{0} \)

= undefined
h. \( \sin \left( -\frac{11\pi}{2} \right) = \sin \left( \frac{3\pi}{2} \right) = 1 \)

i. \( \frac{\tan \left( \frac{5\pi}{6} \right) - 2\cos(2\pi)}{4\sin \left( -\frac{3\pi}{2} \right)} = \frac{-\tan \left( \frac{3\pi}{6} \right) - 2\cos(0)}{4\sin \left( \frac{3\pi}{2} \right)} = \frac{-\frac{\sqrt{3}}{3} - 2 \cdot 0}{-4 \cdot 1} = \frac{-\frac{\sqrt{3}}{3}}{4} = \frac{-\sqrt{3}}{12} - \frac{1}{2} \)

\( \tan \left( \frac{\pi}{8} \right) = \frac{\sin \left( \frac{3\pi}{8} \right)}{\cos \left( \frac{3\pi}{8} \right)} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3} \)
Question 46: Find the perimeter of a sector of a circle with central angle $\theta = \frac{11\pi}{6}$ and radius $r = 4$

a. $8 + \frac{22\pi}{3}$

b. $4 + \frac{22\pi}{3}$

c. $8 + \frac{44\pi}{3}$

d. $\frac{22\pi}{3}$

e. None of above

Question 10

A particle is moving on the perimeter of a circle with radius $r = 8$ with angular speed of $\frac{\pi}{6}$ radians per second. After completing 3 full rotations. What is the length of the total distance the particle traveled?

a. $4\pi$

b. $8\pi$

c. $24\pi$

d. $48\pi$

e. None of the above
Section 4.4: Trigonometric Expressions and Identities

We can manipulate expressions with trig functions using the same techniques we use when manipulating polynomials or rational functions. These techniques include distributing, collecting like terms, putting everything over a common denominator, and factoring.

An identity is an equation that is true for all value(s) of the variable. Sometimes, you can use trig identities to help you simplify trig expressions. Here is a list of trig identities we already seen.

\[ \tan(t) = \frac{\sin(t)}{\cos(t)} \quad \cot(t) = \frac{\cos(t)}{\sin(t)} \]

Reciprocal Identities:

\[
csc(t) = \frac{1}{\sin(t)}, \quad \sin(t) \neq 0 \\
sec(t) = \frac{1}{\cos(t)}, \quad \cos(t) \neq 0 \\
\cot(t) = \frac{1}{\tan(t)}, \quad \tan(t) \neq 0
\]

Pythagorean Identities:

\[
\sin^2(t) + \cos^2(t) = 1 \\
\tan^2(t) + 1 = \sec^2(t) \\
1 + \cot^2(t) = \csc^2(t)
\]

Opposite Angle Identities:

\[
\sin(-t) = -\sin(t) \\
\cos(-t) = \cos(t) \\
\tan(-t) = -\tan(t) \\
\csc(-t) = -\csc(t) \\
\sec(-t) = \sec(t) \\
\cot(-t) = -\cot(t)
\]

Example 1: Simplify.

a. \[ \tan(x)csc(x) \]

\[ = \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\sin(x)} = \csc(x) \]

b. \[ \frac{\sin(x)}{\cos(x) - 1} \cdot \frac{\cos(x) + 1}{\cos(x) + 1} \]

\[ = \frac{\sin(x)(\cos(x) + 1)}{\cos^2(x) - 1} = \frac{\sin(x)(\cos(x) + 1)}{1 - \sin^2(x)} = \frac{\cos(x) + 1}{\sin(x)} \]

\[ = -\cot(x) - \csc(x) \]
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c. \[ \frac{\csc^2(\theta) - 1}{1 - \sin^2(\theta)} \]

d. \[ \frac{\sec^2(\theta)}{\tan(\theta) + \cot(\theta)} \]

e. \[ (\csc(\theta) - \cot(\theta))(\sec(\theta) + 1) \]

f. \[ \frac{\tan x - \cot x}{\tan x + \cot x} + 1 \]
Proving Identities

Some helpful hints when proving identities:

1. Start with the “ugliest” side.
2. Get common denominators.
3. Convert everything to sine and cosine (this frequently works, but sometimes makes things worse).
4. If you see $1 - \sin \theta$, try multiplying by $1 + \sin \theta$.
   If you see $1 + \cos \theta$, try multiplying by $1 - \cos \theta$.
5. If you get stuck working from one side, try starting with the other.

Example 2: Prove the following identities.

a. $\csc x \tan x \cos = 1$

b. $\sec \beta - \cos \beta = \tan \beta \sin \beta$

c. $\frac{\sin \alpha \cos \alpha}{1 - \cos^2 \alpha} = \cot \alpha$
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d. \[ \frac{\cot A (1 + \tan^2 A)}{\tan A} = \csc^2 A \]

e. \[ \frac{\tan x}{1 + \sec x} + \frac{1}{\tan x} = \csc x \]

f. \[ \sin^4 x - \cos^4 x = 2 \sin^2 x - 1 \]