Section 2.1: Linear and Quadratic Functions

A linear function is a function that can be written in the form $f(x) = mx + b$ ($m$ = slope and $b$ = $y$-intercept). Its graph is a straight line.

Recall: If a line passes through $(x_1, y_1)$ and $(x_2, y_2)$ its slope $m$ is given by

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{"\text{rise}"}{"\text{run}"}$$

The sign of the slope indicates whether the line is an increasing or a decreasing function. The slope of any horizontal line is 0 and the slope of any vertical line is undefined.

Two Forms for an Equation of a Line:

1. $y = mx + b$ (slope-intercept form)
2. $y - y_1 = m(x - x_1)$ (point-slope form)

Note: We are working with functions, so we want $y$ on the left-hand side by itself. This means we need to rearrange our equations into slope-intercept form so we can write

$$f(x) = mx + b$$

Example 1: Write an equation for a linear function whose $x$-intercept is 4 and the $x$-intercept for the inverse is 2.
Example 2: Suppose the point $(1, -2)$ is on the graph of a linear function and the point $(4, 3)$ is on the inverse function’s graph. What is the equation for both the inverse and the function?

Parallel Lines and Perpendicular Lines

- Two lines are parallel if and only if their slopes are the same. $(m_1 = m_2)$
- Two lines are perpendicular if and only if their slopes are negative reciprocals of each other.

\[ m_1 = -\frac{1}{m_2} \text{ or also } m_1 m_2 = -1 \]

Example 3: Write the equation of the linear function $f(x)$ that passes through the point $(-4, 5)$ and is parallel to the line $4x + 3y = 7$.

Example 4: Find the linear function $f$ such that $f(2) = 5$ and the graph of $f$ is perpendicular to the line $y = -\frac{1}{14}x - \frac{6}{7}$.
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Quadratic Functions

A quadratic function is a function which can be written in the form \( f(x) = ax^2 + bx + c \), where \( a, b, \) and \( c \) are real numbers and \( a \) is not equal to zero.

The Standard Form of a Quadratic Function

Every quadratic function also known as a parabola is written as \( f(x) = ax^2 + bx + c \) or can be written in standard form: \( f(x) = a(x - h) + k \). The vertex is the point \((h, k)\). The axis of symmetry is the equation \( x = h \).

For a quadratic function, \( f(x) = ax^2 + bx + c \) or \( f(x) = a(x - h)^2 + k \)

- The graph opens up if \( a > 0 \).
- If \( |a| > 1 \), the parabola is narrower; if \( |a| < 1 \) the parabola is wider.

\[
\begin{align*}
\text{The vertex is the turning point of the parabola. If the parabola opens upward the function has a} \\
\text{minimum value (y-value). If the parabola opens downward the function has a maximum value (y-value).}
\end{align*}
\]

The vertex is the turning point of the parabola. If the parabola opens upward the function has a minimum value (y-value). If the parabola opens downward the function has a maximum value (y-value).

The axis of symmetry is a line through the vertex that divides the graph in half.

The vertex of a parabola whose equation is \( f(x) = ax^2 + bx + c \) is

\[
\left( -\frac{b}{2a}, f\left( -\frac{b}{2a}\right) \right)
\]

And the axis of symmetry is \( x = -\frac{b}{2a} \).
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You should be able to identify the following:

- Direction the graphs opens (upwards or downwards)
- Whether the function has a maximum or a minimum
- y-intercept
- coordinates of the vertex
- equations of the axis of symmetry
- maximum or minimum

**Example 5:** Write \( f(x) = -2x^2 + 12x - 15 \) in standard form. Find the

a. vertex

b. axis of symmetry

c. maximum or minimum value

**Example 6:** Sketch the graph of \( f(x) = 3x^2 + 6x + 7 \) by finding the six features of the function.
Example 7: Find the quadratic function such that the axis of symmetry is $x = -2$, the y-intercept is -6 and there is only one x-intercept.