Section 4.4: Trigonometric Expressions and Identities

We can manipulate expressions with trig functions using the same techniques we use when manipulating polynomials or rational functions. These techniques include distributing, collecting like terms, putting everything over a common denominator, and factoring.

An **identity** is an equation that is true for all value(s) of the variable. Sometimes, you can use trig identities to help you simplify trig expressions. Here is a list of trig identities we all ready seen.

\[
\tan(t) = \frac{\sin(t)}{\cos(t)} \quad \cot(t) = \frac{\cos(t)}{\sin(t)}
\]

**Reciprocal Identities:**

\[
csc(t) = \frac{1}{\sin(t)}, \quad \sin(t) \neq 0 \quad \sec(t) = \frac{1}{\cos(t)}, \quad \cos(t) \neq 0 \quad \cot(t) = \frac{1}{\tan(t)}, \quad \tan(t) \neq 0
\]

**Pythagorean Identities:**

\[
\sin^2(t) + \cos^2(t) = 1 \quad \tan^2(t) + 1 = \sec^2(t) \quad 1 + \cot^2(t) = \csc^2(t)
\]

**Opposite Angle Identities:**

\[
\sin(-t) = -\sin(t) \quad \cos(-t) = \cos(t) \\
\tan(-t) = -\tan(t) \quad \csc(-t) = -\csc(t) \\
\sec(-t) = \sec(t) \quad \cot(-t) = -\cot(t)
\]

**Example 1:** Simplify.

a. \(\tan(x)csc(x)\)

b. \(\frac{\sin(x)}{\cos(x) - 1}\)
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c. \[ \frac{\csc^2(\theta) - 1}{1 - \sin^2(\theta)} \]

d. \[ \frac{\sec^2(\theta)}{\tan(\theta) + \cot(\theta)} \]

e. \[ (\csc(\theta) - \cot(\theta))(\sec(\theta) + 1) \]

f. \[ \frac{\tan x - \cot x}{\tan x + \cot x} + 1 \]
Proving Identities

Some helpful hints when proving identities:

1. Start with the “ugliest” side.
2. Get common denominators.
3. Convert everything to sine and cosine (this frequently works, but sometimes makes things worse).
4. If you see $1 - \sin \theta$, try multiplying by $1 + \sin \theta$.
   If you see $1 + \cos \theta$, try multiplying by $1 - \cos \theta$.
5. If you get stuck working from one side, try starting with the other.

Example 2: Prove the following identities.

a. $\csc x \tan x \cos = 1$

b. $\sec \beta - \cos \beta = \tan \beta \sin \beta$

c. $\frac{\sin \alpha \cos \alpha}{1 - \cos^2 \alpha} = \cot \alpha$
d. \[ \frac{\cot A(1 + \tan^2 A)}{\tan A} = \csc^2 A \]

e. \[ \frac{\tan x}{1 + \sec x} + \frac{1}{\tan x} = \csc x \]

f. \[ \sin^4 x - \cos^4 x = 2 \sin^2 x - 1 \]