Opposite Angle Identities

The identities shown below are called **opposite-angle identities**. They tell us that the sine, tangent, cosecant and cotangent are **odd** functions and cosine and secant are **even** functions.

\[
sin(-t) = -\sin(t) \quad \csc(-t) = -\csc(t) \\
\tan(-t) = -\tan(t) \quad \cot(-t) = -\cot(t) \\
\cos(-t) = \cos(t) \quad \sec(-t) = \sec(t)
\]

**Example 1:** Use the opposite-angle identities to evaluate the following.

a. \(\cos\left(\frac{-\pi}{3}\right)\)

b. \(\tan\left(\frac{-\pi}{4}\right)\)

**Example 2:** Simplify. \(\cot(-t)\sec(-t)\)

**Example 3:** Write an equivalent form: \(\csc(-6t) - \sec(-8t)\)

**Example 4:** Suppose \(\sin(t) = -\frac{2}{3}\) and \(\pi < t < \frac{3\pi}{2}\). Find \(\tan(t)\) and \(\sec(t)\).
Example 5: Suppose $\tan(t) = -\sqrt{3}/2$ and $\pi/2 < t < \pi$. Find $\csc(t)$ and $\cos(t)$.

**Periodicity**

The circumference of the unit circle is $2\pi$. Thus, if we start with a point $P$ on the unit circle and travel a distance of $2\pi$ units, we arrive back at the same point $P$. That means that the arc lengths of $t$ and $t + 2\pi$ as measured from the point $(1, 0)$ give the same terminal point on the unit circle. Thus, we have the following identities.

$$\sin(t + 2k\pi) = \sin(t) \quad \cos(t + 2k\pi) = \cos(t)$$

So,

$$\csc(t + 2k\pi) = \csc(t) \quad \sec(t + 2k\pi) = \sec(t)$$

Like the sine and cosine functions, the tangent and cotangent functions also repeat themselves at intervals of lengths $2\pi$. In addition, the tangent and cotangent functions also repeat themselves at intervals of shorter length, namely $\pi$. This, we have the following identities.

$$\tan(t + k\pi) = \tan(t) \quad \cot(t + k\pi) = \cot(t)$$

For all real numbers $t$ and all integers $k$.

Example 6: Evaluate $\sin\left(-\frac{20\pi}{3}\right)$

Example 7: Evaluate $\cot\left(\frac{15\pi}{6}\right)$
Example 8: Evaluate $\frac{\cos\left(\frac{19\pi}{2}\right) \tan\left(\frac{21\pi}{4}\right)}{\cos(8\pi)}$

Example 9: Evaluate $\cot\left(\frac{15\pi}{4}\right) + \frac{\sin\left(\frac{10\pi}{3}\right)}{\cos\left(\frac{17\pi}{6}\right)}$