Section 5.2: Graphs of the Sine and Cosine Functions

A Periodic Function and Its Period

A nonconstant function $f$ is said to be periodic if there is a number $p > 0$ such that $f(x + p) = f(x)$ for all $x$ in the domain of $f$. The smallest such number $p$ is called the period of $f$.

The graphs of periodic functions display patterns that repeat themselves at regular intervals.

Amplitude

Let $f$ be a periodic function and let $m$ and $M$ denote, respectively, the minimum and maximum values of the function. Then the amplitude of $f$ is the number $\frac{M - m}{2}$

Example 1: Specify the period and amplitude of the given function

![Graph of a periodic function](image)

Now let’s talk about the graphs of the sine and cosine functions.

Recall: $\sin(\theta + 2\pi) = \sin(\theta)$ and $\cos(\theta + 2\pi) = \cos(\theta)$

This means that after going around the unit circle once ($2\pi$ radians), both functions repeat. So the period of both sine and cosine is $2\pi$. Hence, we can find the whole number line wrapped around the unit circle.

Since the period of the sine function is $2\pi$, we will graph the function on the interval $[0, 2\pi]$, since the rest of the graph will repeat itself.
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Let’s take a look at Sine

**Example 2**
Sine: \( f(x) = \sin x \)

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The big picture:

Since the period of the cosine function is \( 2\pi \), we will graph the function on the interval \([0, 2\pi]\), since the rest of the graph will repeat itself.

So let’s take a look at the Cosine function.
Example 3

Cosine: \( f(x) = \cos x \)

\[ \begin{align*}
\text{Domain:} & \quad \text{________________} \\
\text{Range:} & \quad \text{________________} \\
\text{Period:} & \quad \text{________________} \\
\text{Amplitude:} & \quad \text{_________} \\
\text{x-intercepts:} & \quad \text{________________} \\
\text{y-intercept:} & \quad \text{________________}
\end{align*} \]

The big picture:

\[ \begin{align*}
\text{Note:} & \quad \text{The graphs of } y = \sin(x) \text{ and } y = \cos(x) \text{ are exactly the same shape. The only difference is to get the graph of } y = \cos(x), \text{ simply shift the graph of } y = \sin(x) \text{ to the left } \frac{\pi}{2} \text{ units.}
\end{align*} \]

In fact \( \sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta) \)
Using the fact that $\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$. These graphs will be translations, reflections, “stretches”, and “squishes” of $y = \sin(x)$ and $y = \cos(x)$.

For the following functions:

$$y = A\sin(Bx - C) \quad \text{and} \quad y = A\cos(Bx - C)$$

Amplitude $= |A|$ (Note: Amplitude is always positive.) If $A$ is negative, that means an $x$-axis reflection.

$$\text{Period} = \frac{2\pi}{B}$$

Translation in horizontal direction (called the phase shift) $= \frac{C}{B}$

We’ll ask you to learn the shape of the graph and just graph five basic points, the $x$ and $y$ intercepts and the maximum and the minimum.

One complete cycle of the sine curve includes three $x$-intercepts, one maximum point and one minimum point. The graph has $x$-intercepts at the beginning, middle, and end of its full period.

One complete cycle of the cosine curve includes two $x$-intercepts, two maximum points and one minimum point. The graph has $x$-intercepts at the second and fourth points of its full period.

Key points in graphing these functions are obtained by dividing the period into four equal parts.

**Example 4:** Give the amplitude, period, and phase shift for the following functions:

a. $f(x) = 2\cos\left(\pi x + \frac{2\pi}{3}\right)$
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b. \( f(x) = 3 \sin \left( \frac{1}{2} x - \frac{\pi}{6} \right) \)

c. \( f(x) = \sin \left( x - \frac{\pi}{6} \right) \)

Example 5: Sketch over one period: \( f(x) = -4 \cos(2\pi x) \)
Example 6: Sketch over one period: \( f(x) = 3\sin(2x) + 1 \)

Example 7: Sketch over one period: \( f(x) = 3\cos\left( x - \frac{\pi}{4} \right) \)
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**Example 8:** Give a function of the form $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$, which could be used to represent the graph. *Note:* these answers are not unique.
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**Example 9:** Give a function of the form $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$, which could be used to represent the graph. *Note:* these answers are not unique.