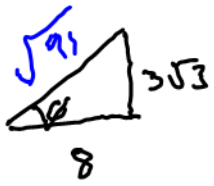
17 ~ 14 MC
\ 3 FR

Math 1330
Test 3 Review
Sections 4.1-4.3, 5.1a, 5.2-5.4

Example 1: Let θ be an acute angle of a right triangle and that $\tan(\theta) = \frac{3}{8}\sqrt{3}$. Find all the rest of the trigonometric functions of θ.

$\tan\phi = \frac{opp}{adj} = \frac{3\sqrt{3}}{8}$



$a^2 + b^2 = c^2$

$(3\sqrt{3})^2 + 8^2 = c^2$

$27 + 64 = c^2$

$91 = c^2$

$\sin\phi = \frac{opp}{hyp} = \frac{3\sqrt{3}}{\sqrt{91}} \cdot \frac{\sqrt{91}}{\sqrt{91}} = \frac{3\sqrt{273}}{91}$

$\cos\phi = \frac{adj}{hyp} = \frac{8}{\sqrt{91}} = \frac{8\sqrt{91}}{91}$

$\csc\phi = \frac{\sqrt{91}}{3\sqrt{3}} = \frac{\sqrt{273}}{9}$

$\sec\phi = \frac{\sqrt{91}}{8}$

$\cot\phi = \frac{8}{3\sqrt{3}} = \frac{8\sqrt{3}}{9}$

Example 2: Let θ be an acute angle of a right triangle and that $\csc(\theta) = \frac{3}{2}\sqrt{5}$. Find all the rest of the trigonometric functions of θ.

$\csc\phi = \frac{hyp}{opp} = \frac{3\sqrt{5}}{2}$

$a^2 + 2^2 = (3\sqrt{5})^2$

$a^2 + 4 = 45$

$a^2 = 41$

$a = \sqrt{41}$

$\sin\phi = \frac{2}{3\sqrt{5}} = \frac{2\sqrt{5}}{15}$

$\cos\phi = \frac{\sqrt{41}}{3\sqrt{5}} = \frac{\sqrt{205}}{15}$

$\tan\phi = \frac{2}{\sqrt{41}} = \frac{2\sqrt{41}}{41}$

$\sec\phi = \frac{3\sqrt{5}}{\sqrt{41}} = \frac{3\sqrt{205}}{41}$

$\cot\phi = \frac{\sqrt{41}}{2}$

Review Test 3

Example 3: A sector of a circle has central angle $\theta = 2\pi/3$ and area $16\pi/3$ ft^2.

a. Find the radius of the circle.

$\text{Area} = \frac{1}{2} r^2 \phi$

$\phi \Rightarrow$ in radians

$\frac{16\pi}{3} = \frac{1}{2} r^2 \cdot \frac{2\pi}{3}$

$\frac{16\pi}{3} = \frac{\pi}{3} r^2$

$\frac{16\pi/3}{\pi/3} = \frac{\pi/3}{\pi/3} r^2$

$r^2 = \frac{16\pi}{3} \cdot \frac{3}{\pi}$

$r^2 = 16$

$r = 4$ ft

b. Find the length of the sector for the problem above.

Length $s = r\phi$

$= 4\left(\frac{2\pi}{3}\right) = \frac{8\pi}{3}$ ft

Example 4: Find the area of the sector with a central angle of $\theta = 240°$ and a $r = 6$ cm

$\text{Area} = \frac{1}{2} r^2 \phi$

$= \frac{1}{2} (6)^2 \left(\frac{4\pi}{3}\right)$

$= \frac{1}{2} \cdot 36 \cdot \frac{4\pi}{3}$

$= 24\pi$

$240° \cdot \frac{\pi}{180°} = \frac{240\pi}{180}$

$= \frac{4\pi}{3}$

Example 5: A car has wheels with a 10 cm radius. If each wheel's rate of turn is 3 revolutions per second

a. What is the angular speed?

3 rev/sec 1 rev = 2π

$\omega = \frac{\phi}{t} = \frac{3 \cdot 2\pi}{\text{sec}} = 6\pi$ / sec

b. How fast is the car moving in units of cm/sec, (linear spead)?

$v = r \cdot \omega$

$= 10\text{cm} \cdot 6\pi/\text{sec}$

$= \boxed{60\pi \frac{\text{cm}}{\text{sec}}}$

Review Test 3

Example 6: Find three angles, two positive and one negative, that are coterminal with the given angle: $\frac{4\pi}{3}$

$\alpha + 2\pi$

$\alpha + 4\pi$

$\alpha - 2\pi$

$\frac{4\pi}{3} + 2\pi = \frac{4\pi}{3} + \frac{6\pi}{3} = \frac{10\pi}{3}$

$4\pi/3 + 4\pi = \frac{4\pi}{3} + \frac{12\pi}{3} = \frac{16\pi}{3}$

$\frac{4\pi}{3} - 2\pi = \frac{4\pi}{3} - \frac{6\pi}{3} = -\frac{2\pi}{3}$

Example 7: Evaluate the following.

$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ $\quad \tan\left(\frac{-5\pi}{6}\right) = \frac{\sqrt{3}}{3}$ $\quad \cot\left(\frac{2\pi}{3}\right) = \frac{\cos(\frac{2\pi}{3})}{\sin(\frac{2\pi}{3})} = \frac{-1/2}{\sqrt{3}/2} = -\frac{\sqrt{3}}{3}$

$\cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ $\quad \sec\left(\frac{5\pi}{6}\right) = \frac{1}{\cos(\frac{5\pi}{6})} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$ $\quad \csc\left(\frac{\pi}{3}\right) = \frac{1}{\sin(\frac{\pi}{3})} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Example 8: Simplify

a. $\sin\left(\frac{14\pi}{3}\right) = \sin\left(\frac{2\pi}{3} + 4\pi\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\sin(t + 2k\pi) = \sin(t)$

b. $\tan\left(\frac{-16\pi}{3}\right) = -\tan\left(\frac{16\pi}{3}\right) = -\tan\left(\frac{\pi}{3} + 5\pi\right) = -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$

$\tan(t + \pi k) = \tan(t)$

c. $4\cos\left(\frac{34\pi}{6}\right) + \cot\left(\frac{17\pi}{4}\right)$

$4\cos\left(\frac{4\pi}{6} + 5\pi\right) = 4\cos\left(\frac{10\pi}{6} + 4\pi\right) = 4\cos\left(\frac{10\pi}{6}\right)$

$= 4\cos\left(\frac{5\pi}{3}\right) = 4\left(\frac{1}{2}\right) = 2$

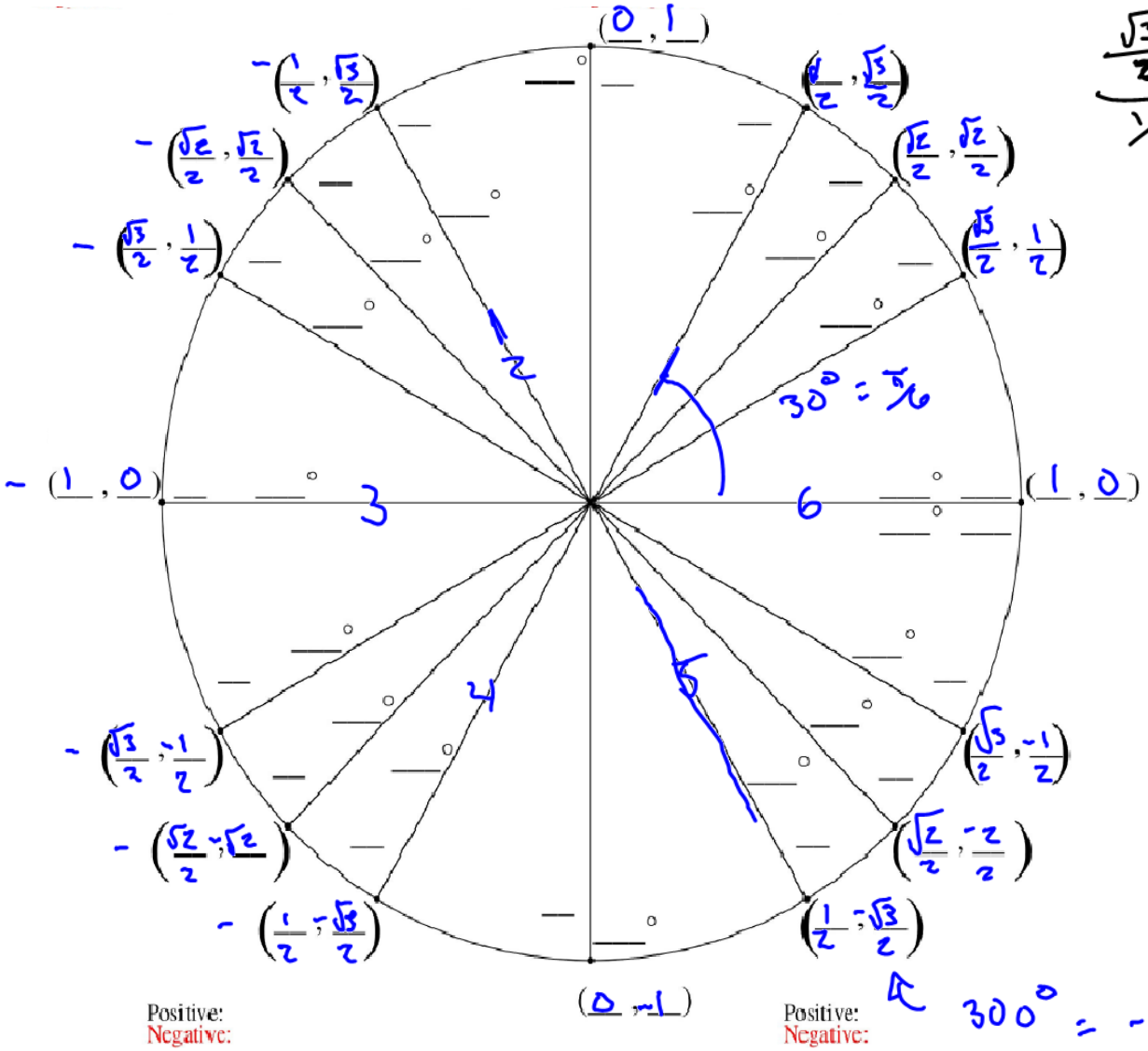
$\cot\left(\frac{17\pi}{4}\right) = \cot\left(\frac{\pi}{4} + 4\pi\right) = \cot\left(\frac{\pi}{4}\right)$

$= 1$

$2 + 1 = \boxed{3}$

Review Test 3

Example 9: Fill in the unit circle



Example 10: Evaluate

$\sin^{-1}\left(-\frac{1}{2}\sqrt{3}\right) = -\frac{\pi}{3}$

$\tan^{-1}(-1) = -\frac{\pi}{4}$

$\cot^{-1}\left(\sqrt{3}\right)$ ← $\frac{x}{y} = \frac{\sqrt{3}/2}{1/2}$ → $\frac{\pi}{6}$

$\sec^{-1}\left(\frac{2\sqrt{3}}{3}\right) = \frac{1}{x} = \frac{\pi}{6}$

$\csc^{-1}\left(\sqrt{2}\right) = \frac{1}{y} = \frac{\pi}{4}$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\frac{2\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2 \cdot \cancel{3}}{\cancel{3} \cdot \sqrt{3}}$

$= \frac{2}{\sqrt{3}} \Rightarrow \frac{\sqrt{3}}{2}$ ← Look up

$\frac{\sqrt{2}}{1} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{2}$

← Look up

Review Test 3

Example 11: Evaluate

$$\cot\left[\cos^{-1}\left(-\frac{3}{7}\right)\right]$$

$\cos(\alpha) = \frac{Adj}{hyp} = \frac{-3}{7}$

$a^2 + (-3)^2 = 7^2$

$a^2 + 9 = 49$

$a^2 = 40$

$a = 2\sqrt{10}$

$\cot(\alpha) = \frac{\cos(\alpha)}{\sin(\alpha)} = \frac{Adj}{Opp} = \frac{-3}{2\sqrt{10}}$

$$\boxed{-\frac{3\sqrt{10}}{20}}$$

Example 12: Find the amplitude, period and phase shift.

$4\cos\left(\frac{1}{3}x\right)$

Amp: 4

Period: $\frac{2\pi}{B} = \frac{2\pi}{1/3} = 6\pi$

PS: 0

$-\frac{1}{2}\sin\left(\frac{2\pi}{3}x + \frac{3\pi}{2}\right)$

Amp: $\frac{1}{2}$

$Bx - C$

Period: $\frac{2\pi}{2\pi/3} = 2\pi \cdot \frac{3}{2\pi} = 3$

PS. $= \frac{C}{B} = \frac{\frac{-3\pi}{2}}{\frac{2\pi}{3}} = \frac{-3\pi}{2} \cdot \frac{3}{2\pi} = \frac{-9}{4}$

$6\sin\left(\frac{1}{5}\pi x + \pi\right)$

Amp: 6

$Bx - C$

Period: $\frac{2\pi}{1/5\pi} = 2\pi \cdot \frac{5}{\pi} = 10$

PS: $\frac{C}{B} = \frac{-\pi}{1/5\pi} = -5$

$5\cos\left(\frac{1}{2}\pi x - \pi\right)$

Amp: 5

Period: $\frac{2\pi}{1/2\pi} = 4$

PS: $\frac{C}{B} = \frac{\pi}{1/2\pi} = 2$

Review Test 3

↳ $y = 0$

Example 13: List all the *x*-intercepts for

$$y = 4\cos\left(4x + \frac{1}{3}\pi\right)$$

on the $[-\pi/6, \pi/2]$

$\cos(\alpha) = 0 \quad \alpha = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

$4x + \frac{1}{3}\pi = -\frac{\pi}{2}$

$4x = -\frac{3\pi}{6} - \frac{2\pi}{6}$

$4x = \frac{-5\pi}{6}$

$x = \frac{-5\pi}{24} < \frac{-4\pi}{24} = -\frac{\pi}{6}$

$4x + \frac{\pi}{3} = \frac{\pi}{2}$

$4x = \frac{3\pi}{6} - \frac{2\pi}{6}$

$4x = \frac{\pi}{6}$

$x = \frac{\pi}{24}$

$\pi/4 = \frac{6\pi}{24}$

$4x + \frac{\pi}{3} = \frac{3\pi}{2}$

$4x = \frac{9\pi}{6} - \frac{2\pi}{6}$

$4x = \frac{7\pi}{6}$

$x = \frac{7\pi}{24}$

$\frac{\pi}{2} = \frac{12\pi}{24} < \frac{13\pi}{24}$

Example 14: Write a cosine function with a positive vertical dilation, given the amplitude is 3, the phase shift is 3 to the left, the vertical shift is 2 up, and the period is 2.

$f(x) = A\cos(Bx - C) + D \quad \text{or} \quad A\cos(B(x - C)) + D$

Amp: $3 = A$

PS: -3

VS: $+2$

Per.: $2 = \frac{2\pi}{B}$

$B = \pi$

$3\cos(\pi(x+3)) + 2$

$3\cos(\pi x + 3\pi) + 2$

Review Test 3

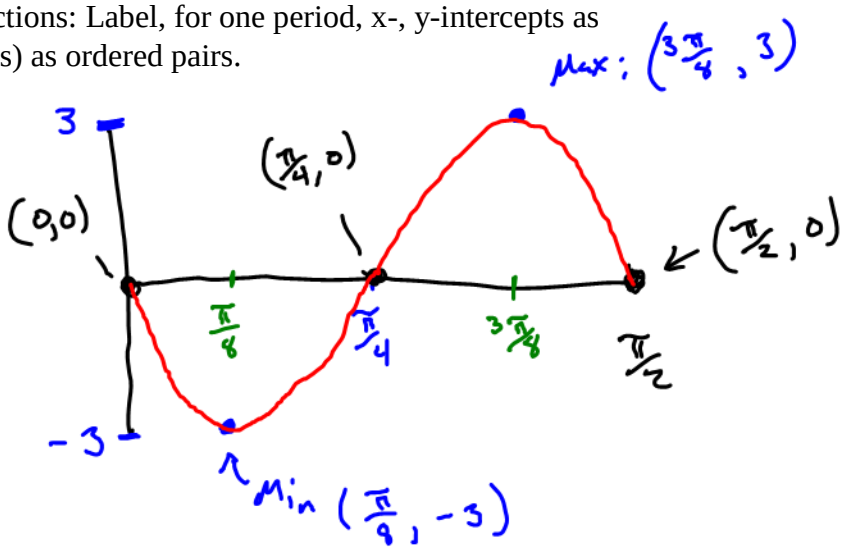
Example 15: Sketch the following functions: Label, for one period, x-, y-intercepts as ordered pairs; max value(s), min value(s) as ordered pairs.

$f(x) = -3\sin(4x)$.

Amp = 3

Period: $\frac{2\pi}{4} = \frac{\pi}{2}$

PS = 0



$\frac{\frac{\pi}{4} + 0}{2} = \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8}$

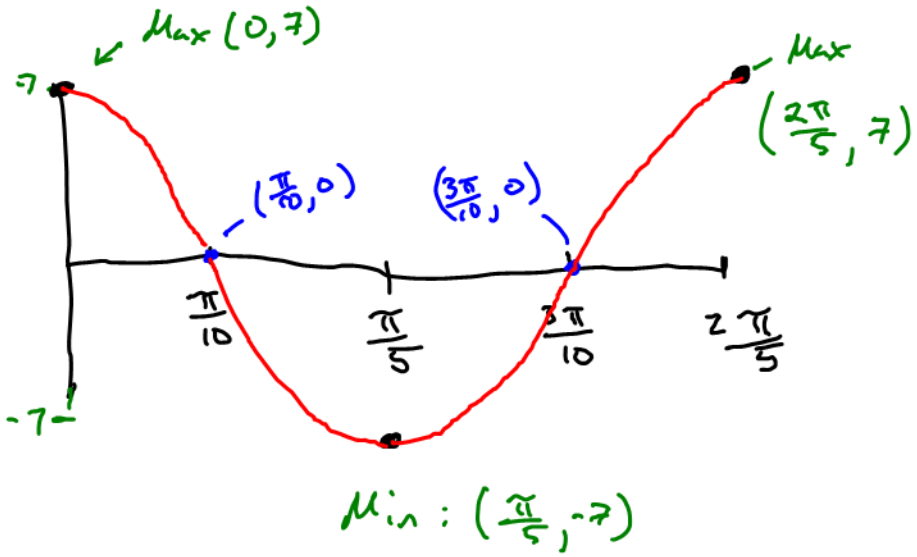
$\frac{\frac{\pi}{2} + \frac{\pi}{4}}{2} = \frac{\frac{2\pi}{4} + \frac{\pi}{4}}{2} = \frac{\frac{3\pi}{4}}{2} = \frac{3\pi}{8}$

$g(x) = 7\cos(5x)$

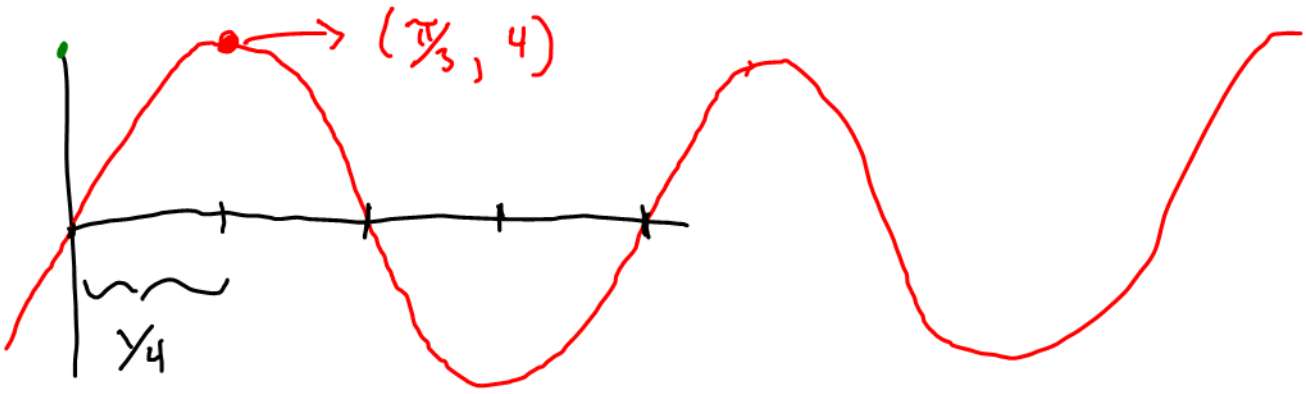
Amp: 7

Period = $\frac{2\pi}{5}$

P.S. = 0



$\frac{\frac{2\pi}{5}}{2} = \frac{2\pi}{10}$



$A\sin(Bx - C) + D$

↑ No PS

$A\cos B(x - C) + D$

Max at $y = 4$
Min $y = -4$
$\Rightarrow$ Amp $= 4 \Rightarrow A = 4$

$A = 4$

P.S. $= \frac{\pi}{3} = C$

Period $= \frac{\pi}{3} \times 4 = \frac{4\pi}{3} = \frac{2\pi}{B}$

$\frac{4\pi}{3} B = 2\pi$

$B = \frac{2\pi}{1} \cdot \frac{3}{4\pi} = \frac{3}{2}$

$4\sin(\frac{3}{2}x)$

Period $= \frac{\pi}{3} \times 4 = \frac{4\pi}{3}$

$\frac{4\pi}{3} = \frac{2\pi}{B}$

$B = \frac{3}{2}$

$D = 0$

$4\cos\frac{3}{2}(x - \frac{\pi}{3}) + 0$

$4\cos(\frac{3}{2}x - \frac{3\pi}{6})$

$4\cos(\frac{3}{2}x - \frac{\pi}{2})$