Section 1.1 - Review

Prerequisites are topics you should have mastered before you enter this class. This section is intended as a quick review of these topics.

An Introduction to Functions

Let A and B be two nonempty sets. A function from A to B is a rule of correspondence that assigns to each element in A exactly one element in B. Here A is called the domain of the function and the set B is called the range of the function.

The Domain of a Function is defined as the set of all possible inputs allowed for that function.

To determine the domain of a function, start with all real numbers and then eliminate anything that results in zero denominators or even roots of negative numbers.

The domain of any **polynomial function** is $(-\infty,\infty)$ or all real numbers.

The domain of any **rational function**, where both the numerator and the denominator are polynomials, is all real numbers except for the values of x for which the denominator equals 0.

The domain of any **radical function** with even index is the set of real numbers for which the radicand is greater than or equal to 0. The domain of any radical function with odd index is $(-\infty,\infty)$.

Example 1: Find the domain: $f(x)=\sqrt{x-3}$

Example 2: Find the domain: $g(x)=\dfrac{4x^2+7}{x^2-4}$

Example 3: Find the domain: $k(x) = 4x^2 - 8x + 11$

Linear Functions

General Equation of a Line is in the form $Ax + By + C = 0$

Slope Intercept Form: $y = mx + b$, where m is the slope and b is the is the y-intercept of the line.

Point -Slope Form: $y - y_1 = m(x - x_1)$, where m is the slope and passes through the point (x_1, y_1) is given

When the m = 0, you have a **horizontal line**.
When the m = undefined, you have a **vertical line**.

Example 4: Suppose the slope of a line is $\frac{1}{3}$ and the line passes through the point $(-3, 7)$.
Write the equation of the line in slope-intercept form ($y = mx + b$).

Rational Functions

A rational function is a function of the form $f(x)=\dfrac{P(x)}{Q(x)}$, where P and Q are polynomial functions and $Q(x)\neq 0$. You'll need to be able to find the following features of the graph of a rational function and then use the information to sketch the graph

- Domain
- Intercepts
- Holes
- Asymptotes(Vertical/Horizontal)
- Behavior near vertical asymptotes

Vertical Asymptotes/Holes:
Factor the numerator and denominator. Look at each factor in the denominator.

- If a factor cancels with a factor in the numerator, then there is a hole where that factor equals zero.
- If a factor does not cancel, then there is a vertical asymptote where that factor equals zero.

Horizontal Asymptotes
Let $f(x)=\dfrac{p(x)}{q(x)}$,

Shorthand: degree of f = deg(f), numerator = N, denominator = D

1. If deg(N) > deg(D) then there is no horizontal asymptote.

2. If deg(N) < deg(D) then there is a horizontal asymptote and it is $y = 0$ (x-axis).

3. If deg(N) = deg(D) then there is a horizontal asymptote and it is $y = \dfrac{a}{b}$, where

   a is the leading coefficient of the numerator.
   b is the leading coefficient of the denominator.

Example 5: Find any hole(s), vertical and/or horizontal asymptotes: $f(x)=\dfrac{x^2+x}{x^2-1}$

Example 6: Find any vertical and/or horizontal asymptotes: $g(x) = \dfrac{x-2}{x^2-4x+3}$

Functions

Example 7: If $f(x) = 2x^2 - 8x$, find:

a. $f(-2)$

b. $f(4+h)$

Difference quotient also called the **average rate of change.**

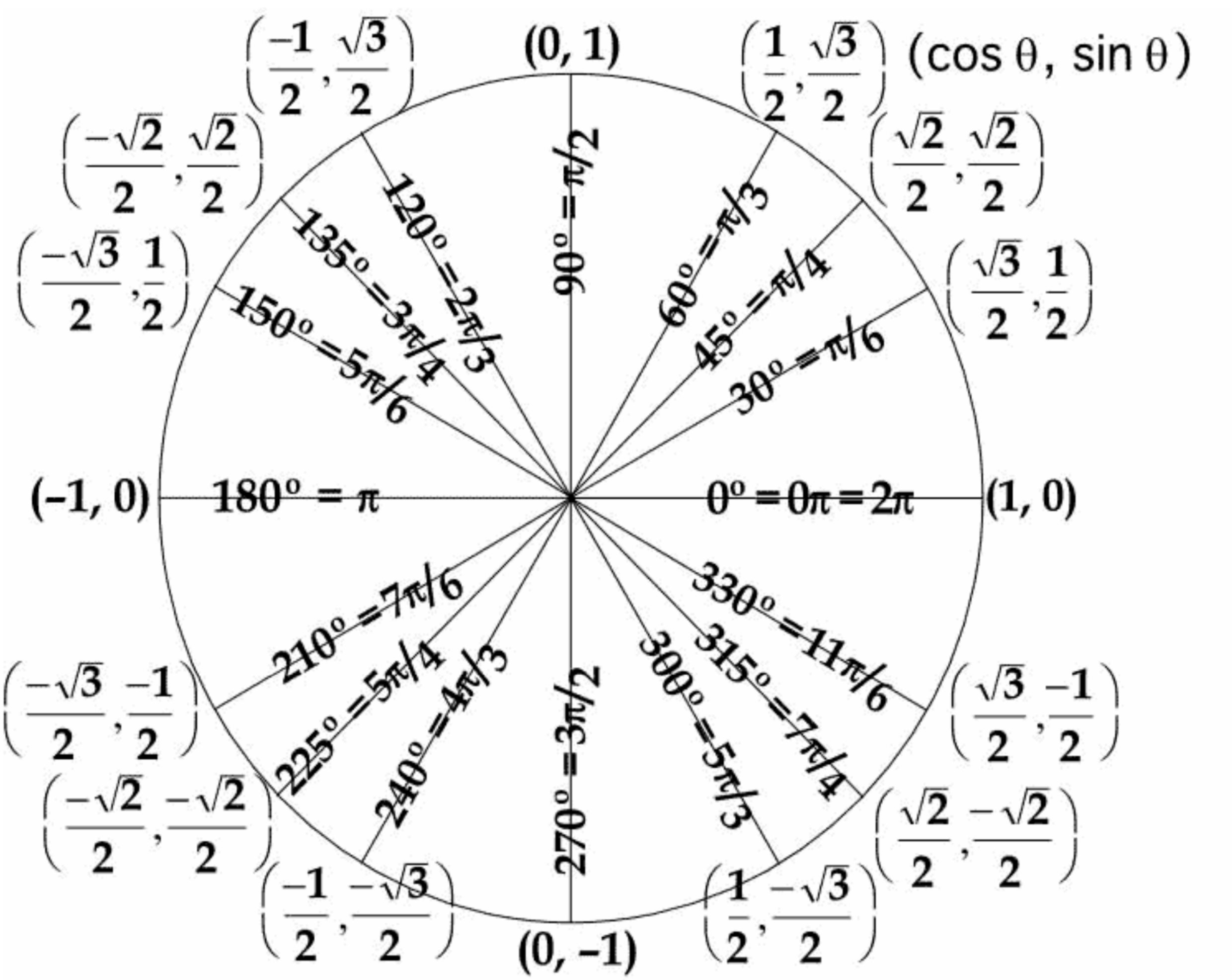
$$m = \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}$$

A function is **even** if $f(x) = f(-x)$ for all x in the domain of the function. Even functions are symmetric with respect to the y–axis

A function is **odd** if $f(-x) = -f(x)$ for all x in the domain of the function. Odd functions are symmetric with respect to the origin.

Things to remember for trigonometry

Unit Circle



Quotient Identities:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Reciprocal Identities:

$$\csc(\theta) = \frac{1}{\sin(\theta)},\ \sin(\theta) \neq 0 \qquad \sec(\theta) = \frac{1}{\cos(\theta)},\ \cos(\theta) \neq 0 \qquad \cot(\theta) = \frac{1}{\tan(\theta)},\ \tan(\theta) \neq 0$$

Pythagorean Identities:

$$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \tan^2(\theta) + 1 = \sec^2(\theta) \qquad 1 + \cot^2(\theta) = \csc^2(\theta)$$

Addition Formulas

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \qquad \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B \qquad \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double-Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta}$$