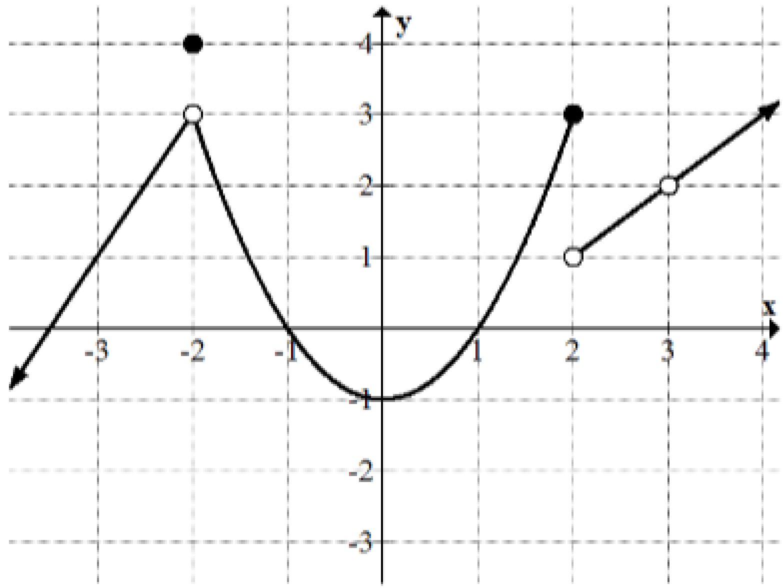
Section 1.2: The Idea of a Limit

Finding a limit amounts to answering the following question: What is happening to the y-value of a function as the x-value approaches a specific target number? If the y-value is approaching a specific number, then we can state the limit of the function as x gets close to the target number.

Example 1: Given the function, $f(x)$.



What value do we expect from the left of x = -2?

What value do we expect from the right of x = -2?

What value do we expect at x = -2?

What value do we expect from the left of x = 2?

What value do we expect from the right of x = 2?

What value do we expect at x = 2?

Informal definition of a Limit: We say that the function *f* has the **limit** *L* as *x* approaches a number *c*, written $\lim_{x \to c} f(x) = L$ if the value $f(x)$ can be made as close to the number *L* as we like by getting *x* sufficiently close to, but not equal to, *c*.

Note the L must be a real value, otherwise the limit fails to exist (DNE =Does Not Exist)

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Example 2: Let's revisit the graph from example 1. Evaluate the following limits.

a. $\lim_{x \to -1} f(x)$

b. $\lim_{x \to 2} f(x)$

c. $\lim_{x \to 3} f(x)$

Notice that when we look for the limit of a function as we approach a specific x-value, we look at the left and right hand side of the graph. If we are only interested in the behavior of a function when we look from one side and not from the other, we are looking at a **one-sided limit.**

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Left-Hand Limit

$\lim_{x \to c^-} f(x) = L$

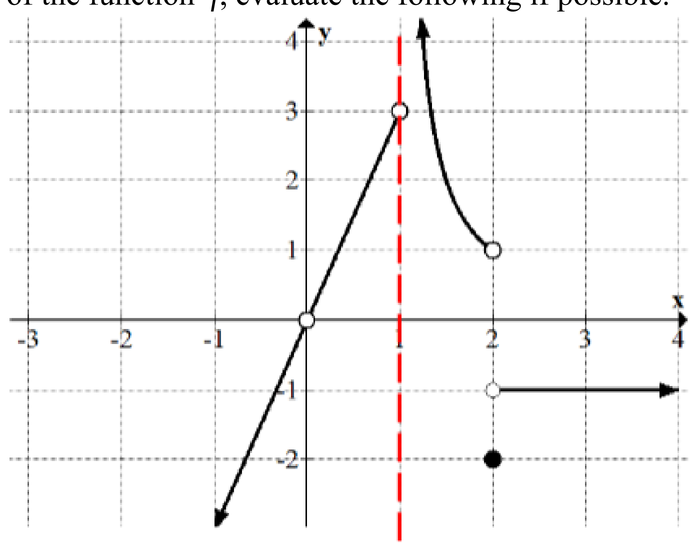
Right-Hand Limit

$\lim_{x \to c^+} f(x) = L$

Which bring us to a more formalized definition.

Let *f* be a function that is defined for all values of x close to *c*, except perhaps at *c* itself. Then $\lim_{x \to c} f(x) = L$ **if and only if** $\lim_{x \to c^-} f(x) = L$ and $\lim_{x \to c^+} f(x) = L$

Example 3: Given the graph of the function *f*, evaluate the following if possible.



$f(0)$ $\lim_{x \to 0^-} f(x)$ $\lim_{x \to 0^+} f(x)$ $\lim_{x \to 0} f(x)$

$f(1)$ $\lim_{x \to 1^-} f(x)$ $\lim_{x \to 1^+} f(x)$ $\lim_{x \to 1} f(x)$

$f(2)$ $\lim_{x\to 2^-} f(x)$ $\lim_{x\to 2^+} f(x)$ $\lim_{x\to 2} f(x)$

Example 4: Evaluate each limit, if it exist. *(Remembering their graphs could help)*

a. $\lim_{x\to 3}(x^2 - 1)$

b. $\lim_{x\to -4^+} \sqrt{x+4}$

c. $\lim_{x\to -4} \sqrt{x+4}$

d. $\lim_{x\to 0} \frac{|x|}{x}$

e. $f(x) = \begin{cases} x^2, & x < 2 \\ 3x, & x \geq 2 \end{cases}$