Section 2.2: Derivatives of Polynomials and Trigonometric Functions

As you saw in several examples in Section 2.1, finding the derivative of a function using the definition is quite tedious. We now introduce some rules that will make our computations much easier.

A constant function is represented by a horizontal line. What is the slope of a horizontal line?

Then the derivative of any constant is zero!

Rule 1: The Derivative of a Constant

$\frac{d}{dx}[k] = 0$, where k is a constant.

Example 1: Find the derivative of $f(x) = -17$.

Rule 2: The Power Rule

$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$ for any real number n

Example 2: Find the derivative of each function

$f(x) = x^5$

$g(x) = x^{-12}$

$h(x) = \sqrt{x}$

$i(x) = x^{\frac{3}{5}}$

Rule 3: Derivative of a Constant Multiple of a Function

$\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$ where k is any real number

Example 3: Find the derivative of each function.

a. $f(x) = 4x^5$

b. $h(x) = \dfrac{2}{3x^4}$

Theorem: Let k be any real number. If f and g are differentiable at x, then so are $f + g$ and $f - g$. Moreover, $(f \pm g)'(x) = f'(x) \pm g'(x)$

Example 4: Find the derivative: $f(x) = -\dfrac{3}{4}x^6 + \dfrac{5}{x^3} + 6\sqrt[3]{x^2} + 13$

Example 5: Find the derivative: $f(x) = \dfrac{3x^5 - 7x^2 + 3}{x^3}$

In some cases, we may want to find all points for which the tangent line to the graph of f is horizontal or equal to a specified number. Set the derivative equal to the given number and solve for x

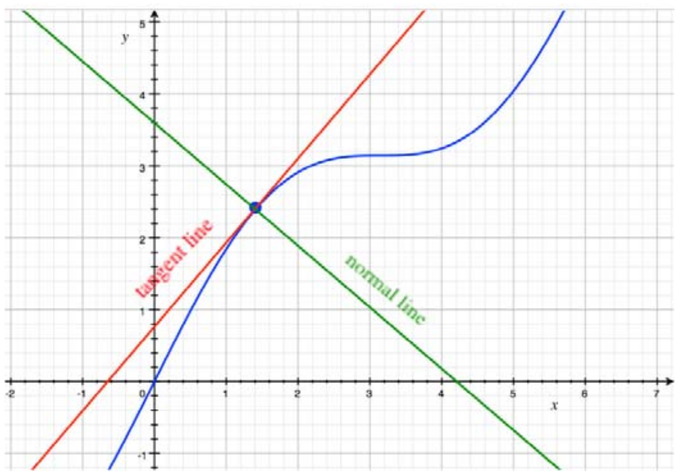
Example 6: Find all x-value(s) on the graph of $f(x) = x^3 + 5x^2$ where the tangent line is horizontal.

Example 7: Find all x-value(s) on the graph of $f(x) = x^2 + 4x + 1$ where $f'(x) = 5$.

Tangent and Normal Lines

We already known what a tangent line is all about.

A normal line to a curve at a particular point is the line through that point and perpendicular to the tangent



Recall from Pre-Algebra the slope of any line perpendicular to a line with slope is the negative reciprocal.

Example 8: Find the equations of the tangent and normal line to $f(x) = 3x^2 + 4x + 2$ at x = -1

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Higher Order Derivatives

$f'(x)$, $f''(x)$, $f'''(x)$, $f^{(4)}(x)$

$\frac{d}{dx}f(x)$, $\frac{d^2}{dx^2}f(x)$, $\frac{d^3}{dx^3}f(x)$, $\frac{d^4}{dx^4}f(x)$,

Example 9: Find $\frac{d^3}{dx^3}f(x)$ for $f(x)=2x^5-3x^3+7x$

Derivatives of the Trigonometric Functions

$(\sin x)'=\cos x$

$(\cos x)'=-\sin x$

$(\tan x)'=\sec^2 x$

$(\csc x)'=-\csc x\cot x$

$(\sec x)'=\sec x\tan x$

$(\cot x)'=-\csc^2 x$

Example 10: Find the slope of the tangent line to the function $f(x)=4\tan x-6\cos x$ at $x=\frac{\pi}{4}$

Example 10: Find all values of x on $[0, 2\pi)$ where the tangent line is horizontal to $f(x) = 6\sqrt{3}\sin x + 18\cos x$.

Now that we know much easier rules for finding derivatives, let's revisit differentiability with piecewise functions.

Example 11: Is this function differentiable?

$$f(x) = \begin{cases} 3x & x > 3 \\ x^2 + x - 3 & x \le 3 \end{cases}$$