Section 2.4: Implicit Differentiation

Let's now say that we wish to find $\frac{dy}{dx}$ for the equation, $x^2 + y^3 - 2xy = 4$. We'd have to first solve for *y*, which is not possible or other times it may be too difficult. To find the derivative of this equation with respect to *x*, we'll use what we call implicit differentiation. *Keep in mind, with these problems, y is an expression in terms of x (but we don't know what y looks like).*

So when taking the derivative of *y* that's in terms of *x*, use the chain rule.

For example, the derivative of y^2 in terms of *x is* $2y \cdot \frac{dy}{dx}$.

Steps for Implicit Differentiation

1. Differentiate both sides of the equation with respect to *x*.
2. Collect all $\frac{dy}{dx}$ terms.
3. Factor $\frac{dy}{dx}$.
4. Solve for $\frac{dy}{dx}$.

In solving in terms of x, take the derivative as usual. In solving in terms of y, use the chain rule.

Example 1: Find $\frac{dy}{dx}$ if $2x^2 + y^3 + 4y^2 = x - 5$.

Example 2: Find $\frac{dy}{dx}$ if $y^3 + 2y^2 - 3y + x = 2$.

Example 3: Find $\frac{dy}{dx}$ if $2\sin x \cos y = 1$.

Example 4: Find $\frac{dy}{dx}$ if $\tan(xy) = 3x + 2y$.

Example 5: Find $\frac{d^2y}{dx^2}$ if $x^2 - y^2 = 3$; Then find $\left.\frac{d^2y}{dx^2}\right|_{\substack{x=2 \\ y=1}}$.

Example 6: Find the slope of the tangent line to the curve at the point (2, 1), give $x^2 + xy + y^2 = 7$

Example 7: Find the equation of the tangent line at $\left(-1/2, -\pi/6\right)$ for $x - \cos(4y) = 0$.

Try this one: Find $\frac{d^2y}{dx^2}$ if $x^2 + y^2 = 36$.