Section 2.4: Implicit Differentiation

Let's now say that we wish to find \( \frac{dy}{dx} \) for the equation, \( x^2 + y^3 - 2xy = 4 \). We’d have to first solve for \( y \), which is not possible or other times it may be too difficult. To find the derivative of this equation with respect to \( x \), we’ll use what we call implicit differentiation. *Keep in mind, with these problems, \( y \) is an expression in terms of \( x \) (but we don’t know what \( y \) looks like).*

So when taking the derivative of \( y \) that’s in terms of \( x \), use the chain rule.

For example, the derivative of \( y^2 \) in terms of \( x \) is \( 2y \cdot \frac{dy}{dx} \).

Steps for Implicit Differentiation

1. Differentiate both sides of the equation with respect to \( x \).
2. Collect all \( \frac{dy}{dx} \) terms.
3. Factor \( \frac{dy}{dx} \).
4. Solve for \( \frac{dy}{dx} \).

*In solving in terms of \( x \), take the derivative as usual. In solving in terms of \( y \), use the chain rule.*

**Example 1:** Find \( \frac{dy}{dx} \) if \( 2x^2 + y^3 + 4y^2 = x - 5 \).
Example 2: Find $\frac{dy}{dx}$ if $y^3 + 2y^2 - 3y + x = 2$.

Example 3: Find $\frac{dy}{dx}$ if $2 \sin x \cos y = 1$. 
**Example 4:** Find \( \frac{dy}{dx} \) if \( \tan(xy) = 3x + 2y \).

**Example 5:** Find \( \frac{d^2y}{dx^2} \) if \( x^2 - y^2 = 3 \); Then find \( \frac{d^2y}{dx^2} \bigg|_{x=2} \) when \( y = 1 \).
Example 6: Find the slope of the tangent line to the curve at the point (2, 1), given $x^2 + xy + y^2 = 7$.

Example 7: Find the equation of the tangent line at $\left(-\frac{1}{2}, \frac{-\pi}{6}\right)$ for $x - \cos(4y) = 0$.

Try this one: Find $\frac{d^2y}{dx^2}$ if $x^2 + y^2 = 36$. 