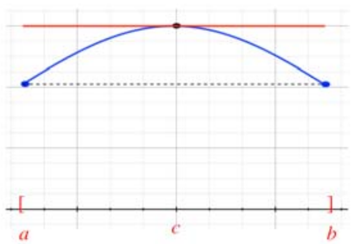
Math1431 Section 3.2

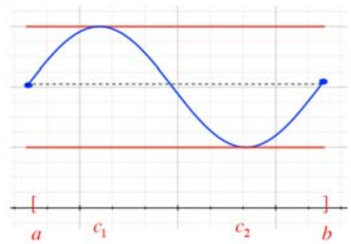
Section 3.2: Rolle's Theorem and the Mean Value Theorem

Rolle's Theorem

Suppose that f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a)=f(b)$, then there is at least one number c in (a,b) for which $f'(c)=0$.

The essence of Rolle's theorem may be seen on these pictures:





Rolle's Theorem sometimes states:

Suppose that f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a)=f(b)=0$, then there is at least one number c in (a,b) for which $f'(c)=0$. That is, Rolle's theorem tells us that between two roots of *f* , there must be a root to f '.

Example 1: Verify that the Rolle's Theorem applies to the function $f(x)=x^2-x-20$ over $[-3,4]$. Then find all points in this interval that satisfy Rolle's Theorem.

- If *f* continuous on [-3,4]?
- If *f* differentiable on (-3, 4)?
- *f*(-3) = *f*(4) =

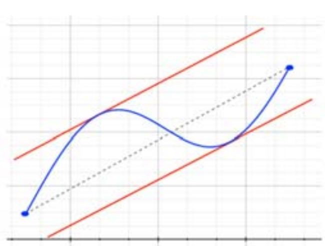
Example 2: Verify that the Rolle's Theorem applies to the function $f(x)=cos(2x)$ over $[0,\pi]$. Then find all points in this interval that satisfy Rolle's Theorem.

- If *f* continuous on $[0,\pi]$?
- If *f* differentiable on $(0,\pi)$?
- *f*(0) = *f*(π) =

The mean-value theorem is a generalization of the Rolle's Theorem.

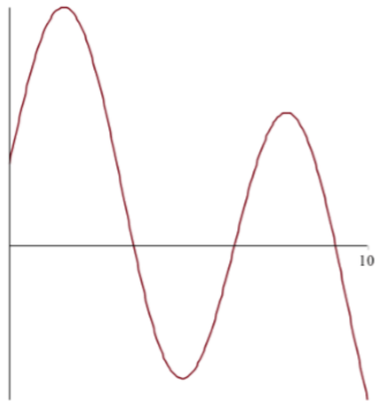
The Mean-Value Theorem

Suppose that f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). There is at least one number c in (a,b) for which $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.



The conclusion of the Mean Value Theorem states that there exists a point *c* in the interval (*a*, *b*) such that the tangent line is parallel to the line passing through (*a*, *f*(*a*)) and (*b*, *f*(*b*)).

Example 3: At how many points between 0 and 10 does the function satisfy the Mean Value theorem?



Example 4: Verify that the Mean Value Theorem applies to the function $f(x) = x^3 + x - 4$ over [-1, 2]. Then find all points in this interval that satisfy Mean Value Theorem.

- If *f* continuous on [-1,2]?
- If *f* differentiable on (-1,2)?

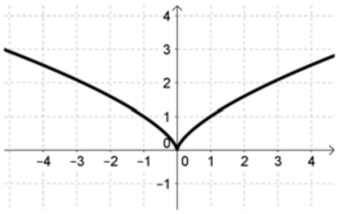
Example 5: Does the Mean Value Theorem apply to the function $f(x) = \frac{x+2}{x}$ over [-1, 2].

- If *f* continuous on [-1,2]?
- If *f* differentiable on (-1,2)?

Example 6: Verify that the Mean Value Theorem applies to the function $f(x) = \sqrt{16 - x^2}$ over [0, 4]. Then find all points in this interval that satisfy Mean Value Theorem.

- If *f* continuous on [0,4]?
- If *f* differentiable on (0,4)?

Example 7: Does the Mean Value Theorem apply to the function $f(x) = x^{2/3}$ over [-1, 1]?



Be careful with any form of $f(x) = |x|$ *as well, as this function has a sharp corner!*