Section 3.3: Increasing and Decreasing Functions

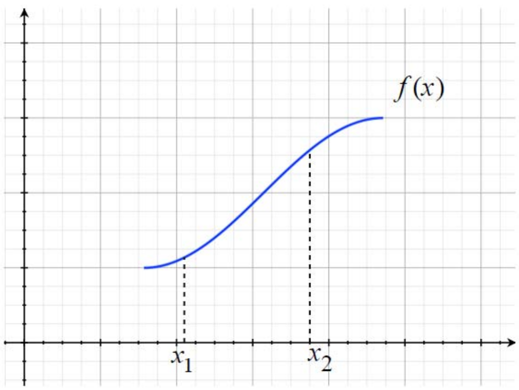
Let f be a function whose domain includes an interval I.

We say that f is **increasing** on I if for every two numbers x_1, x_2 in I,

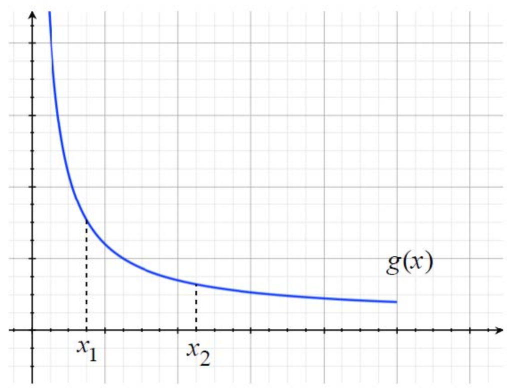
$$x_1 < x_2 \text{ implies that } f(x_1) < f(x_2).$$

We say that f is **decreasing** on I if for every two numbers x_1, x_2 in I,

$$x_1 < x_2 \text{ implies that } f(x_1) > f(x_2).$$

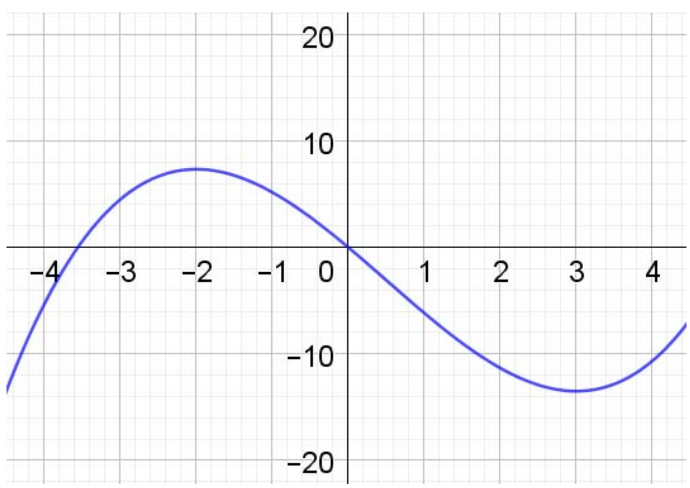


f is increasing



g is decreasing

Example 1: Given the graph of a polynomial function below, give the interval(s) of increasing and decreasing.



Increasing: Decreasing:

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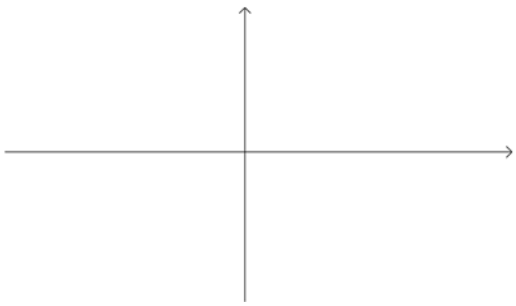
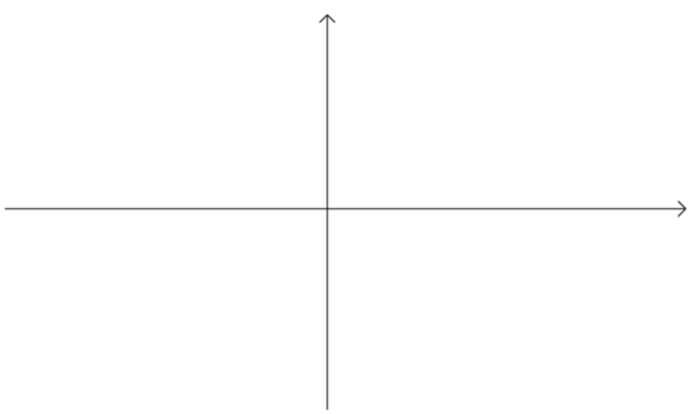
One way we can find intervals of increase and decrease is to graph the function.

Example 2: Given $f(x) = 5|x-2|+1$, when is this function increasing? When is it decreasing?



Increasing: Decreasing:

Example 3: Given $f(x) = \begin{cases} x^2+1, & x<0 \\ 2x, & 0 \le x \le 5 \\ -x, & 5<x \end{cases}$, when is this function increasing? When is it decreasing?



Increasing: Decreasing:

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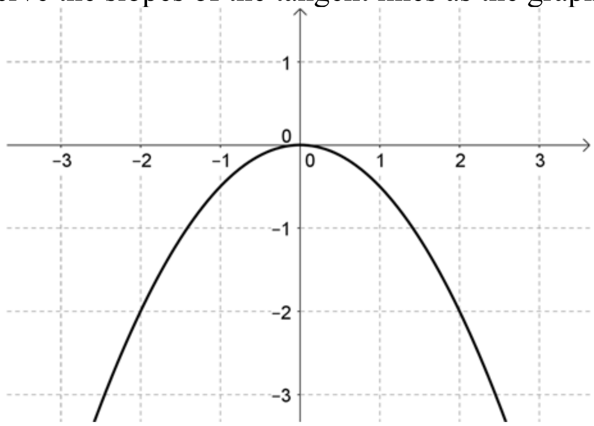
Let's use the graph below to observe the slopes of the tangent lines as the graph increases and decreases.



Over the intervals where the function is increasing, the tangent lines have positive slope. On the other hand, over the intervals of decrease, the tangent lines have negative slope.

Theorem: Suppose that f is differentiable on the interior of an interval I and continuous on all of I.

- If $f'(x) > 0$ for all x in I, then f increases on I.
- If $f'(x) < 0$ for all x in I, then f decreases on I.

Finding intervals where a Function is Increasing or Decreasing

1. Find all values of x for which $f'(x) = 0$ or $f'(x)$ is undefined. Identify the intervals determined by these points.

2. Choose a test point c in each interval found in Step 1 and determine the sign of f' in that interval.
   a. Wherever $f'(c) > 0$, then the function f is increasing on that interval.
   b. Wherever $f'(c) < 0$, then the function f is decreasing on that interval.

Example 4: Given $f(x) = 6x^5 - 40x^3 + 10$, when is this function increasing? When is it decreasing?

Increasing: Decreasing:

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Example 5: Given $f(x) = (x-5)^{2/3}$, determine the intervals over which the function is increasing/decreasing.

$$f'(x) = \frac{2}{3(x-5)^{1/3}}$$

Roots?

Undefined?

Increasing:

Decreasing:

Example 6: Given $f(x) = \frac{\sin x}{1+\cos x}$, when is this function increasing on $(0,\pi)$? When is it decreasing on $(0,\pi)$?

$$f'(x) = \frac{1}{1+\cos x}$$

Roots?

Undefined?

Increasing:

Decreasing: