Section 3.3

Increasing and Decreasing Functions
Intuitively, where is $f$ increasing?
Intuitively, where is $f$ decreasing?
In plain terms, a function is increasing if, as $x$ moves to the right, its graph moves up, and is decreasing if its graph moves down.

A function is strictly monotonic on an interval if it is either increasing on the entire interval or decreasing on the entire interval.

Since $f'(a)$ gives us the slope of the tangent line to $f(x)$ at $x = a$, it follows that:

- where $f'(x)$ is positive, $f(x)$ is increasing
- and
- where $f'(x)$ is negative, $f(x)$ is decreasing.
In math terms........
Definition:

\[ f \] is increasing over an interval \( I \)
if and only if \( f(a) < f(b) \)
For all \( a, b \) in \( I \) with \( a < b \).

Theorem: A function \( f \) is increasing on an interval \( I \) provided \( f \) is continuous and \( f'(x) > 0 \) at all but finitely many values in \( I \).

What property does the derivative have on this interval?
Definition:

A function $f$ is decreasing over an interval $I$ if and only if

$$f(a) > f(b)$$

For all $a, b$ in $I$ with $a < b$.

Theorem: A function $f$ is decreasing on an interval $I$ provided $f$ is continuous and $f'(x) < 0$ at all but finitely many values in $I$.

What property does the derivative have on this interval?
Definition of Critical Number:

The numbers c in the domain of a function $f$ for which either $f'(c) = 0$ or $f'(c)$ does not exist, are called the critical numbers of $f$.

The terms critical points and critical values are also used.

Example 1. Find the critical numbers of $f(x) = 3x^4 - 4x^3$. 
Example 2. Find the critical numbers of \( f(x) = \frac{x-1}{x+3} \).

Example 3. Find the critical number(s) for \( f(x) = \left(x^2 - 36\right)^{\frac{1}{3}} \).
Theorem: Test for Increasing or Decreasing Functions

Let \( f \) be a function that is continuous on the closed interval \([a, b]\) and differentiable on the open interval \((a, b)\).

1. If \( f'(x) > 0 \) for all \( x \) in \((a, b)\), then \( f \) is increasing on \([a, b]\).

2. If \( f'(x) < 0 \) for all \( x \) in \((a, b)\), then \( f \) is decreasing on \([a, b]\).

3. If \( f'(x) = 0 \) for all \( x \) in \((a, b)\), then \( f \) is constant on \([a, b]\).

To find intervals on which a continuous function is increasing or decreasing:

1. Locate the critical numbers to determine test intervals.
2. Determine the sign of \( f'(x) \) at one value in each interval.
3. Using the previous theorem, determine if the function is increasing or decreasing on the interval.
Determine the intervals of increase and/or decrease for each of the following.

Example 4. \( g(x) = \frac{x^2 + 1}{x^2 - 1} \)
Example 5. \( f(x) = \frac{1}{3} x^3 + \frac{5}{2} x^2 + 6x - 3 \)
Example 6. \( g(x) = \frac{2x}{x^2 - 4} \)