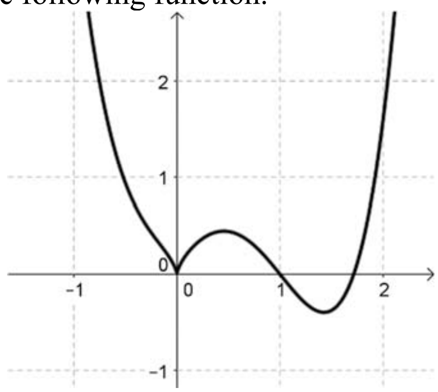
Section 3.4: Extreme Values

Local Extreme Values

Suppose that f is a function defined on open interval I and c is an interior point of I. The function f has a local **minimum** at $x = c$ if $f(c) \leq f(x)$ for all x in I (that is, for all x sufficiently close to c). The function f has a local **maximum** at $x = c$ if $f(c) \geq f(x)$ for all x in I (that is, for all x sufficiently close to c).

In general, if f has a local minimum or maximum at $x = c$, we say that $f(c)$ is a **local extreme value** of f.

Identify the local extreme values for the following function.



This graph suggests that local maxima or minima occur at the points where the tangent line is horizontal or where the function is not differentiable, and this is true.

If c is in the domain of a function f for which $f'(c) = 0$ or $f'(c)$ does not exist, then c is called **critical point for** f.

First Derivative Test

Let c be a critical number of a function f that is continuous on an open interval I containing c.

1. If $f'(c)$ changes from negative to positive at c, then $f(c)$ is a local minimum of f.
2. If $f'(c)$ changes from positive to negative at c, then $f(c)$ is a local maximum of f.

So you want to make a sign chart when using the first derivative test.

Example 1: Find any critical points(value) and local extreme points: $f(x) = x^3 - 3x^2 - 24x + 32$

Domain of $f(x)$:

Find when $f'(x) = 0$:

Find when $f'(x) =$ undefined:

Apply First Derivative Test:

$f'(x)$:

Local Extreme Points

Local Max:

Local Min:

Example 2: Find any critical points(value) and local extreme points $f\left(x\right)=64x^2+\dfrac{54}{x}-2$.

Domain of $f(x)$:

Find when $f'(x)=0$:

Find when $f'(x)=$ undefined:

Apply First Derivative Test:

$f'(x)$:

Local Extreme Points

Local Max:

Local Min:

Example 3: Find any critical points(value) and local extreme points: $f\left(x\right)=\dfrac{x^2}{\left(x-4\right)}$

Domain of $f(x)$:

$$f\,'(x)=\frac{x(x-8)}{\left(x-4\right)^2}$$

Find when $f\,'(x)=0$:

Find when $f\,'(x)=$ undefined:

Apply First Derivative Test:

$f\,'(x)$:

Local Extreme Points

Local Max:

Local Min:

Example 4: Find any critical points(value) and local extreme points: $f\left(x\right)=\left(x-3\right)^{2/5}$

Domain of $f(x)$:

$$f'(x)=\frac{2}{5\left(x-3\right)^{3/5}}$$

Find when $f'(x)=0$:

Find when $f'(x)=$ undefined:

Apply First Derivative Test:

$f'(x)$:

Local Extreme Points

Local Max:

Local Min:

Math1431 Section 3.4

Sometimes it is difficult to study the sign of the derivative function. For some cases, it may be easier to use the following test:

The Second Derivative Test

Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

- If $f''(c) > 0$, then $f(c)$ is a local minimum.
- If $f''(c) < 0$, then $f(c)$ is a local maximum.
- If $f''(c) = 0$, then the test fails. In such cases, you can use the First Derivative Test.

Example 5: Find any critical points(value) and local extremum on $\left(0, \frac{3\pi}{4}\right)$ using the second derivative test/

$f(x) = 2\sin x + \cos(2x)$

Find when $f'(x) = 0$:

Find when $f'(x) =$ undefined:

$f''(x) =$

Apply Second Derivative Test:

Local Extreme Points

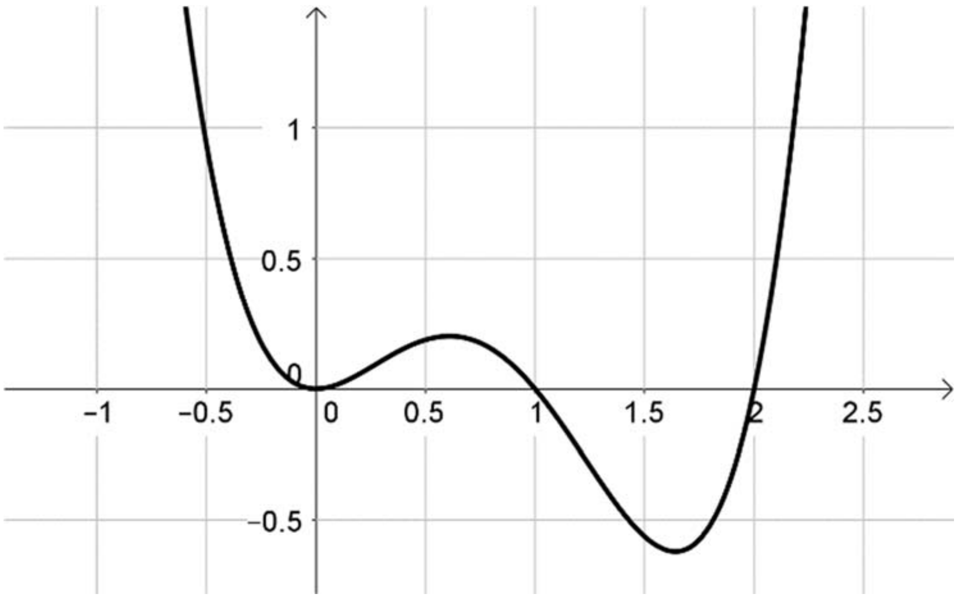
Local Max:

Local Min:

Math1431 Section 3.4

What can we say about f given the graph of f'? Given the graph of f' we can find the critical numbers of *f* and where it's increasing or decreasing.

Example 6: Below is the graph of the derivative of a polynomial function *f*. Which of the following statements is/are true or false?



a. The critical numbers for *f* are 0, 1 and 2.

b. The function *f* has two minimums.

c. The function *f* has one maximum.

d. The function *f* is decreasing over one interval.

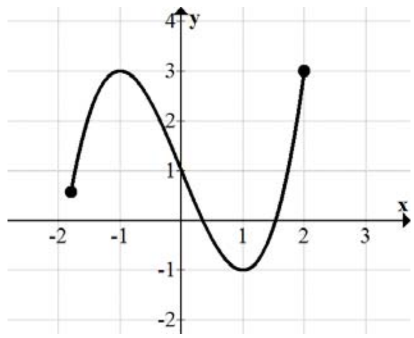
Math1431 Section 3.4

Absolute Extreme Values

Let c be a point in the domain of f ; c may be an interior point or an endpoint.

We say that

- f has an **absolute minimum** at c if $f(x) \geq f(c)$ for all x in the domain of f ,
- f has an **absolute maximum** at c if $f(x) \leq f(c)$ for all x in the domain of f .



If f takes on an absolute extreme value, then it does so at a critical number or at an endpoint.

Finding the absolute minimum and maximum values of a continuous function defined on a closed bounded interval $[a,b]$:

1. Find the critical points for f in the interval (a,b).
2. Evaluate the function at each of these critical points and at the endpoints.
3. The smallest of these computed values is the absolute minimum value, and the largest is the absolute maximum value of f .

Example 7: Find the absolute extrema of $f(x) = x^3 + 3x^2 - 1$ over [-3, 2].

Example 8: Find and classify all absolute extreme values of the function $f(x) = x^{2/3}$ over [1, 8].

Math1431 Section 3.4

Try these:

Given $f(x) = 2x - 2\cos(x)$, when is this function increasing on $[0, 2\pi]$? When is it decreasing on $[0, 2\pi]$?

Given $f(x) = \dfrac{2x}{x^2 - 1}$, when is this function increasing? When is it decreasing?

Given $f(x) = 4\sqrt{x} - 2x$, when is this function increasing? When is it decreasing?

Find any critical points and classify any extrema for: $f(x) = \begin{cases} x+9, & \text{if } -8 \le x < -3 \\ x^2 + x, & \text{if } -3 \le x \le 2 \\ 5x - 4, & \text{if } 2 < x < 5 \end{cases}$

Find and classify all absolute extreme values of the function $f(x) = \tan x - x$ over $\left[-\dfrac{\pi}{3}, \dfrac{\pi}{2}\right)$.