Section 4.2: The Exponential Function

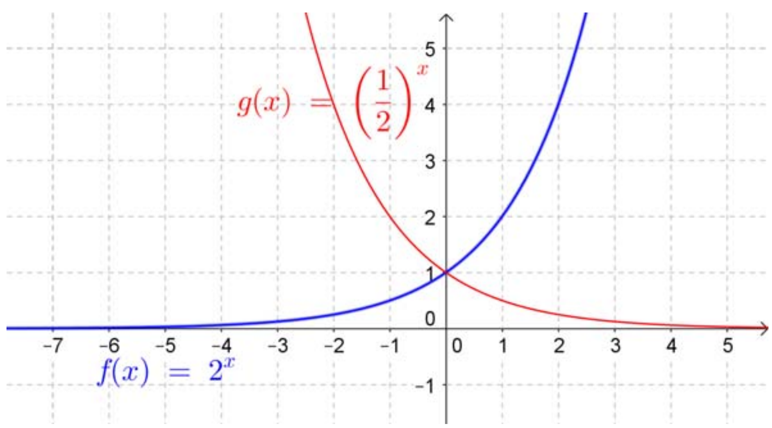
The **exponential function *f* with base *a*** is defined by $f(x) = a^x$ ($a > 0$ and $a \neq 1$) and x is any real number.

If $a = e$ (*the natural base,* $e \approx 2.7183$), then we have $f(x) = e^x$.

- $e^0 = 1$
- $e^1 = e$
- $e^{-1} = \frac{1}{e}$

Graphs

- **If *a* > 1, the graph of** $f(x) = a^x$ looks like (larger a results in a steeper graph):
- **If 0 < *a* < 1, the graph of** $f(x) = a^x$ looks like (smaller a results in a steeper graph):



Their domain is $(-\infty, \infty)$ and range is $(0, \infty)$.

Both graphs have a horizontal asymptote of y = 0 (the x-axis).

Derivatives of Exponential Functions

The exponential function with base "e" has the unique property that it is its own derivative.

$$\frac{d}{dx}\left(e^{x}\right)=e^{x}$$

If the exponent is more than just x, you'll need to use the chain rule to differentiate.

Example 1: Differentiate the following functions.

a. $y=10e^{x}$

b. $g(x)=e^{\sqrt{\sin x}}+e^{10/x}$

c. $f(x)=4x^{3}e^{x^{2}}$ then find equations of the tangent and normal line at $\left(2,32e^{4}\right)$.

Example 2: Let $f\left(x\right)=\dfrac{e^{2x}+e^{-2x}}{e^{x}}$, find $f\,'\left(x\right)$.

Example 3: Let $f\left(x\right)=\dfrac{e^{x}+e^{-x}}{1+e^{2x}}$, find $f\,'\left(x\right)$.

What if the base of the exponential function is NOT *e*?

The derivative of an exponential function is a constant time the function itself.

$$\frac{d}{dx}\left(a^x\right) = a^x \ln a$$

You may need to use the chain rule here too.

Example 4: Differentiate the following functions.

a. $y = 5^x$

b. $f(x) = 4^{x^3+x}$

c. $g(x) = x \cdot 2^{\sin x}$

Example 5: Let $f(x) = 1 - e^{-x}$. Find the intervals where the function is increasing/decreasing/concave up/concave down.

Try this one: For $f(x) = 2 + e^{x\cos(2x)}$, find the equation of the tangent line at x=0: