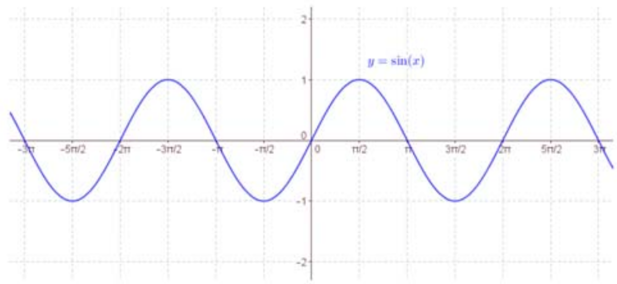
Math 1431 Section 4.4

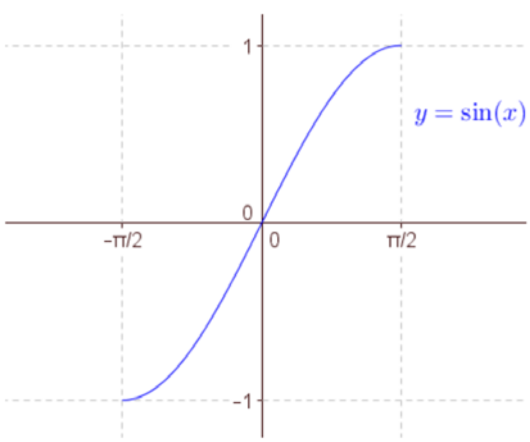
Section 4.4: The Inverse Trigonometric Functions

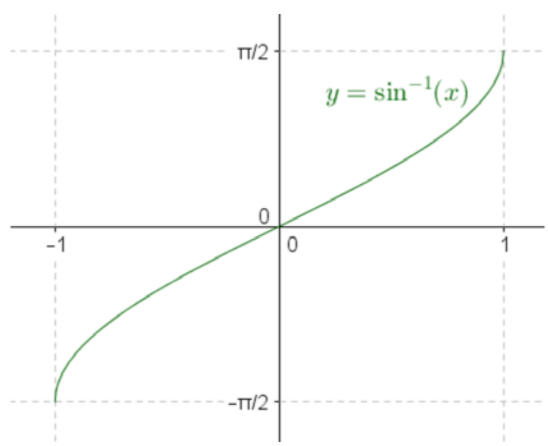
In section 4.1, we learned that in order to have an inverse, a function must be one-to-one. Since trigonometric functions are periodic, they are NOT one-to-one. To define the inverse trig functions, we must restrict the usual domains.

The function sin(x) is graphed below. Notice that this graph does not pass the horizontal line test; therefore, it is not invertible.

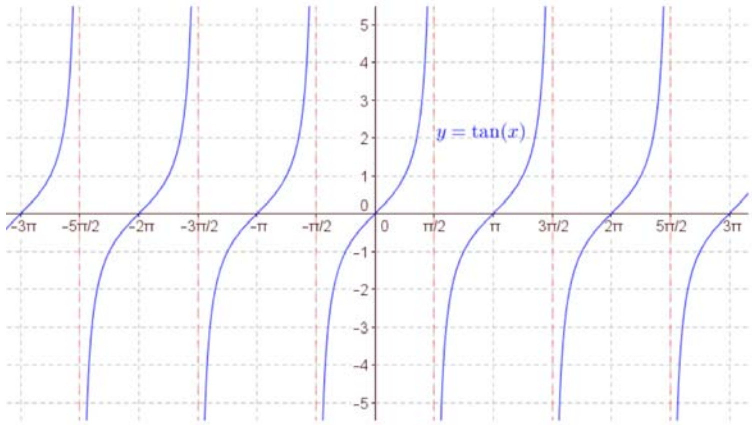


However, if we restrict it from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$ then we have created the **"Restricted" sine function** and it's one-to-one. Since the restricted sine function is one-to-one, it has an inverse $f(x) = \sin^{-1}(x) = \arcsin(x)$.

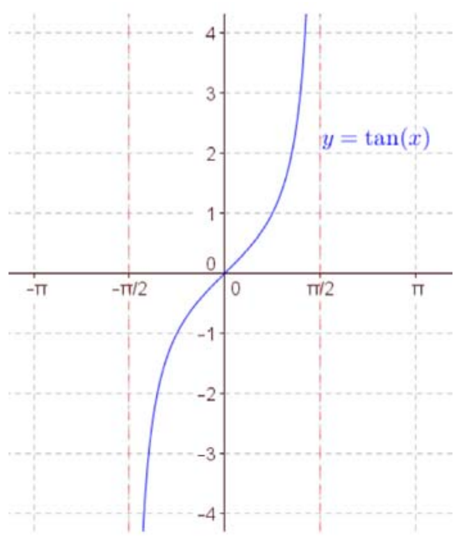


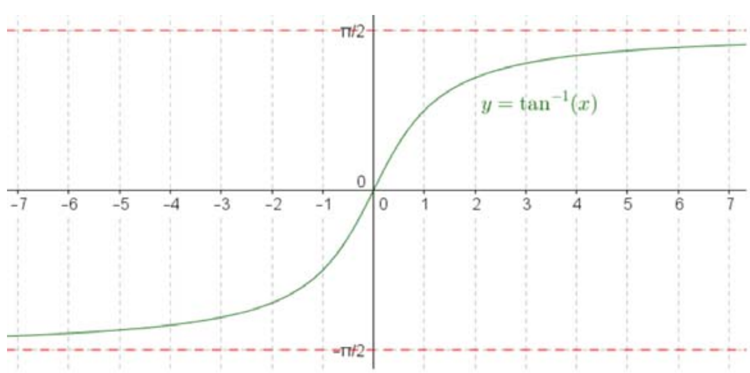


The function tan(x) is graphed below. Notice that this graph does not pass the horizontal line test; therefore, it is not invertible.



However, if we restrict it from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$ then we have created the **"Restricted" tangent function** and it's one-to-one. Since the restricted tangent function is one-to-one, it has an inverse $f(x) = \tan^{-1}(x) = \arctan(x)$.





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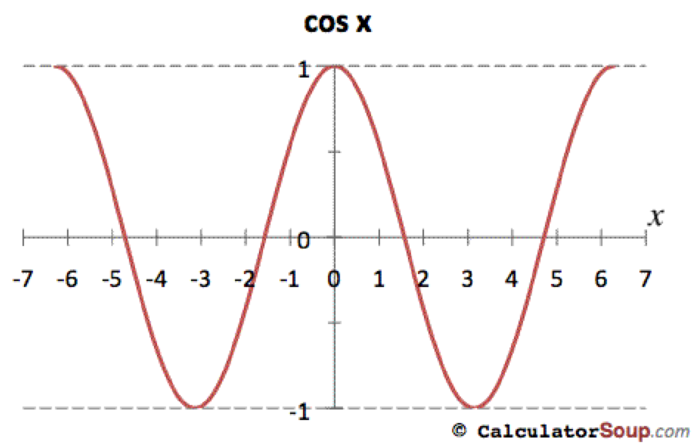
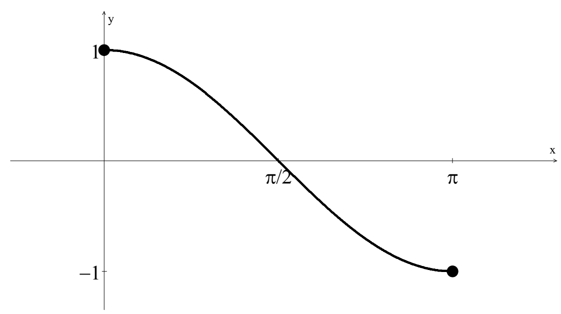
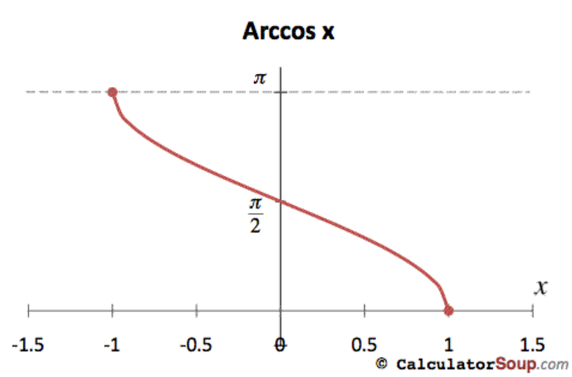
The function cos(x) is graphed below. Notice that this graph does not pass the horizontal line test; therefore, it does not have an inverse.

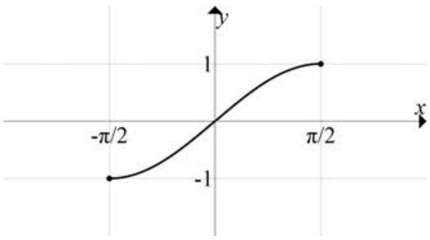


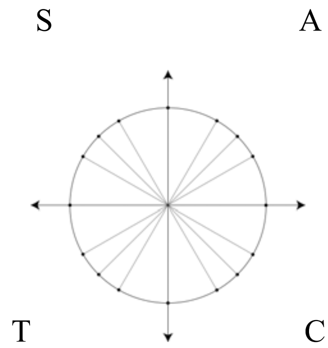
However, if we restrict it from $x = 0$ to $x = \pi$ then we have created the **"Restricted" cosine function** and it's one-to-one. Since the restricted cosine function is one-to-one, it has an inverse $f(x) = \cos^{-1}(x)$.





The restrictions when working with arcsine are: $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ which are angles from **QUADRANTS 1 AND 4**.

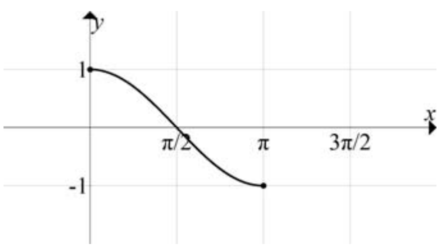


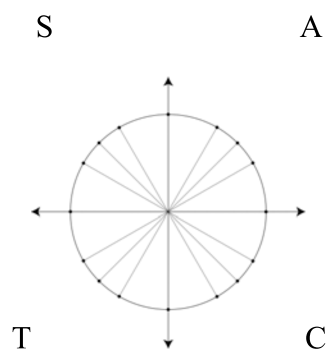


Example 1: Compute $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

The restrictions when working with arccosine are:

$[0,\pi]$ which are angles from **QUADRANTS 1 AND 2**.

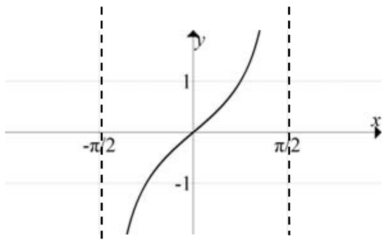


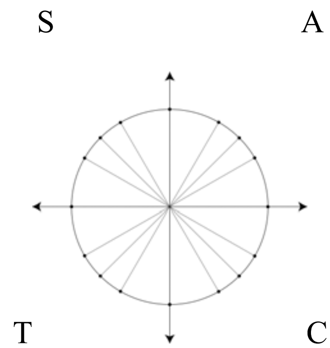


Example 2: Compute $\cos^{-1}\left(\frac{1}{2}\right)$.

The restrictions for arcsec are: $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$. *The restrictions for arccsc are:* $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$

The restrictions when working with arctangent are: $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ which are angles from **QUADRANTS 1 AND 4**.





Example 3: Compute $\arctan\left(\frac{1}{\sqrt{3}}\right)$.

The restrictions for arccot are: $(0, \pi)$

Example 4: Find $\sin^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)$.

Example 5: Find $\cot\left(\tan^{-1}\left(-\sqrt{3}\right)\right)$.

For some problem we'll need to recall the following identities:

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$\cos(2\alpha) = 1 - 2\sin^2\alpha$

$\cos(2\alpha) = 2\cos^2\alpha - 1$

$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$

Example 6: Find $\sin\left(2\arcsin\left(\frac{5}{13}\right)\right)$.

Derivative Formulas (*u* is a function of *x*):

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

Example 7: Differentiate: $y = \cos(\arcsin(2x))$.

Example 8: Differentiate: $y = \sec^{-1}(7x^2)$

Example 9: Differentiate: $\mathrm{f}(x) = e^{\arctan(x)} + \arcsin(\ln x)$

Example 10: Differentiate: $f(x) = 6e^{3x} \arcsin x$

Example 11: Given $g(x) = \arcsin\left(\frac{e^x}{2}\right)$, find the equation for the tangent line to the graph of this function at x = 0.

Example 12: Differentiate: $f(x) = \sqrt{25 - x^2} + 5\arcsin\left(\frac{x}{5}\right)$

Try these:

Find $\tan^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$.

Find $\cos\left(2\arcsin\left(\frac{3}{5}\right)\right)$.

Differentiate:

a. $y = \arcsin(2x^2 + 5x)$

b. $f(x) = \ln(\arctan(3x^2 + 2x))$

c. $g(x) = \frac{x}{\sqrt{36 - x^2}} - \arcsin\left(\frac{x}{6}\right)$

d. $h(x) = \arcsin\left(\frac{e^{6x}}{3}\right)$