Section 5.1: Optimization

Optimization problems (to maximize or minimize):

1. Draw a picture, label it.
2. Determine a function that models the situation and write it in terms of one variable.
3. Find the derivative of the function and its critical numbers so you may optimize it.
4. Once you have the critical points, use the second derivative test to verify that you have an absolute max or an absolute min.

Example 1: Suppose you want to fence in a rectangular shaped region along the straight edge of a river. The side along the river does not need to be fenced. In addition, the region will be subdivided into two parts. The fencing material that will be used for the three sides that are perpendicular to the river costs \$5 per linear foot, and the fencing for the other side costs \$10 per linear foot. Suppose you have \$450 to spend on fencing. Find the dimensions of the total region that will provide the maximum area. What is the maximum area?

1. Determine the function that describes the situation, and write it in terms of one variable (usually x).

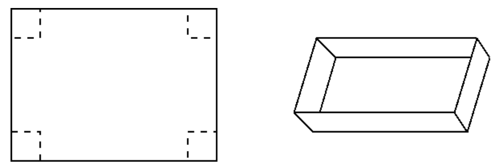
2. Find its derivative and critical numbers.

3. Verify you have a maximum.

4. Dimensions? Max Area?

Example 2: If you cut away equal squares from all four corners of a piece of cardboard and fold up the sides, you will make a box with no top. Suppose you start with a piece of cardboard the measures 3 feet by 8 feet. Find the dimensions of the box that will give a maximum volume. What is the maximum volume?

1. Determine the function that describes the situation, and write it in terms of one variable (usually *x*).



2. Find its derivative and critical numbers.

3. Verify you have a maximum.

4. Dimensions? Max Volume?

Example 3: An open top rectangular box with a square base is to be built to hold 32 cubic feet. What should the dimensions be in order to minimize the cost of material used to build this box?

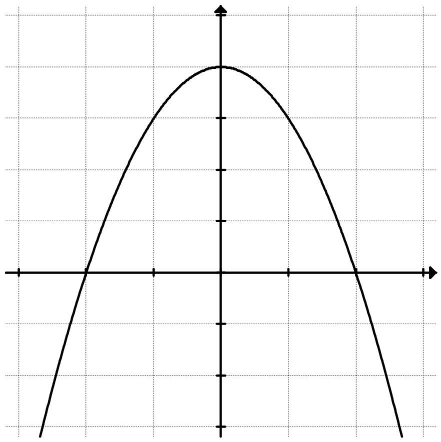
1. Determine the function that describes the situation, and write it in terms of one variable (usually x).

2. Find its derivative and critical numbers.

3. Verify you have a minimum.

4. Dimensions?

Example 4: Find the point(s) on the graph of $y = 4 - x^2$ closest to $(0, 2)$.



Recall: The distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

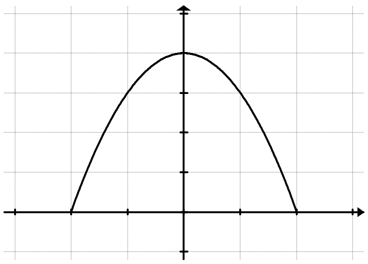
1. Determine the function that describes the situation, and write it in terms of one variable (usually *x*).

2. Find its derivative and critical numbers.

3. Verify you have a minimum.

4. Closets point(s)?

Example 5: Find the largest possible area for a rectangle with base on the x-axis and upper vertices on the curve $y = 4 - x^2$.



1. Determine the function that describes the situation, and write it in terms of one variable (usually *x*).

2. Find its derivative and critical numbers.

3. Verify you have a maximum.

4. Max area?

Example 6: Find A and B such that $y = Ax^{-1/2} + Bx^{1/2}$ has a minimum value of 6 at x=9.

Try this one: A man wishes to have a vegetable garden enclosed by a fence in his backyard. The garden is to be a rectangular area of 60 ft ^{2}. Find the dimensions of the garden that will minimize the amount of fencing material needed.

1. Determine the function that describes the situation, and write it in terms of one variable (usually *x*).

2. Find its derivative and critical numbers.

3. Verify you have a minimum.

4. Dimensions?

Try this one: A closed top rectangular box with a square base will be built to hold 1000 cubic feet. What should the dimensions be in order to minimize the cost of material used to build this box?

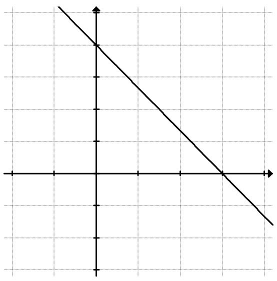
1. Determine the function that describes the situation, and write it in terms of one variable (usually *x*).

2. Find its derivative and critical numbers.

3. Verify you have a minimum.

4. Dimensions?

Try this one: A rectangle sits in the first quadrant with its base on the x-axis and its left side on the y-axis. Its upper right hand corner is on the line passing through the points (0, 4) and (3, 0). What is the largest possible area of this rectangle?



1. Determine the function that describes the situation, and write it in terms of one variable (usually *x*).

2. Find its derivative and critical numbers.

3. Verify you have a maximum.

4. Max area?