Section 5.2: Differentials

The idea here is to use the derivative to approximate certain values. Many other methods exist, but the method here is called **differentials**. This method is easy to use and is quite accurate for small increments.

For $h \neq 0$, we know $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

For small *h*, $f'(x) \approx \frac{f(x+h) - f(x)}{h}$ and from here we get:

$$f'(x) \cdot h \approx f(x+h) - f(x)$$

Solving this equation for $f(x+h)$ we can approximate $f(x+h)$ by applying the formula:

$$f(x+h) \approx f(x) + f'(x) \cdot h$$

The product $f'(x) \cdot h$ is called the *differential* f at *x* with increment h.

Example 1: Given $f'(x) = (x^3 + 5)^{1/5}$ and $f(3) = 2$, use differentials to estimate $f(3.2)$.

Recall: $f(x+h) \approx f(x) + f'(x) \cdot h$

What do you want to approximate? *So, x+h equals?* *Use x equal to?*

Example 2: Use differentials to approximate $\sqrt[3]{25}$.

$f(x+h) \approx f(x) + f'(x) \cdot h$

Example 3: Given that $\ln(2) \approx 0.69$, use differentials to estimate $\ln(2.2)$.

$f(x+h) \approx f(x) + f'(x) \cdot h$

Example 4: Use differentials to approximate $\cos(58^o)$.

$f(x+h) \approx f(x) + f'(x) \cdot h$

Other times, you're simply asked for only the differential $f'(a) \cdot h$.

Example 5: The total cost incurred in operating a certain type of truck on a 500-mile trip, traveling at an average speed of *v* mph, is estimated to be $C(v) = 125 + v + \frac{4500}{v}$ dollars. Find the approximate change in total operating cost when the average speed is increased from 55 mph to 58 mph.

Try this one: Use differentials to approximate $31^{2/5}$.