Section 6.3: Basic Integration Rules

The notation $\int f(x)\ dx$ is used for an antiderivative of f and called an **indefinite integral**.

$$\int f(x)\ dx = F(x) \text{ means } F'(x) = f(x)$$

In general, to find $\int f(x)\ dx$, we find an antiderivative of $f(x)$, say $F(x)$, and then we write the indefinite integral as $\int f(x)\ dx = F(x) + C$. Here, C is called the **constant of integration**.

If given $\int_a^b f(x)\ dx$, this is a **definite integral** and to evaluate we'll use Part 2 of the Fundamental Theorem of Calculus: $\int_a^b f(x)\ dx = F(b) - F(a)$

The Constant Rule for Integrals

$\int k \cdot\ dx = k \cdot x + C$, where k is a constant number.

Example 1: Find of each of the following integrals.

a. $\int 10dx$

b. $\int_1^4 \pi dx$

The Power Rule for Integrals

$\int x^r \; dx = \frac{x^{r+1}}{r+1} + C$, where $r \neq -1$.

Example 2: Find of each of the following integrals.

a. $\int x^4 \; dx$

b. $\int_0^1 \frac{1}{\sqrt{x}} \, dx$

The Constant Multiple of a Function for Integrals

$\int k \cdot f(x)\ dx = k\int f(x)\ dx$, where k is a constant number.

The Sum/Difference of Functions for Integrals

$\int [f(x) \pm g(x)]\ dx = \int f(x)\ dx \pm \int g(x)\ dx$.

Example 3: $\int_0^1 \left(5x^4 + 4x + 7\right) dx$

Example 4: $\int (\sqrt{x}+4)(\sqrt{x}-4)\ dx$

Integrals of Basic Trigonometric Functions:

$\int sin\,x \; dx = -cos\,x + C$

$\int cos\,x \; dx = sin\,x + C$

$\int sec^2 x \; dx = tan\,x + C$

$\int csc^2 x \; dx = -cot\,x + C$

$\int sec\,x\,tan\,x \; dx = sec\,x + C$

$\int csc\,x\,cot\,x \; dx = -csc\,x + C$

Example 5: $\int_{\pi/4}^{\pi/2} \csc x(\sin x + \csc x)dx$

Integral of $\frac{1}{x}$

$$\int \frac{1}{x}\ dx = ln|x| + C$$

Example 6: $\int_1^2 \frac{x^2 - 2}{x}\ dx$

Integrals of Exponential Functions

$$\int e^x\ dx = e^x + C$$

$$\int a^x\ dx = \frac{a^x}{ln\, a} + C\ ,\quad \text{where}\quad a > 0,\ a \neq 1$$

Example 7: $\int \left(2^x + 5e^x\right)\ dx$

Integrals of the Hyperbolic Functions

$\int sinh\, x \; dx = cosh\, x + C$ $\int cosh\, x \; dx = sinh\, x + C$

You may need to use the following definitions as well. $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$ *;* $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$

Example 8: $\int \frac{1}{2}\sinh(x)dx$

Integrals Resulting in Inverse Trigonometric Functions

$\int \frac{1}{\sqrt{1-x^2}} \; dx = arcsin\, x + C$ $\int \frac{1}{1+x^2} \; dx = arctan\, x + C$ $\int \frac{1}{|x|\sqrt{x^2-1}} \; dx = arc\, sec\, x + C$

Example 9: $\int_{1/2}^{\sqrt{2}/2} \frac{2}{\sqrt{1-x^2}}dx$

Other times we're given the derivative and an initial value and we're asked to find the original function.

Example 10: Given $f''(x) = 6x + 2,\ f'(0) = 2,\ f(0) = 10$, find $f(x)$.

Integrating Piece-wise Defined Functions

Example 11: Let $f(x) = \begin{cases} x+2, & -2 \le x \le 0 \\ 2, & 0 < x \le 1 \\ 4-2x, & 1 < x \le 2 \end{cases}$.

Note how the function changes over the specified domain!

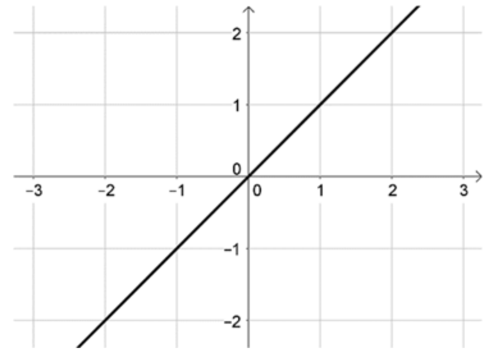
Set-up the integral needed to integrate $\int_{-2}^{2} f(x)dx$.

Integrals Involving Absolute Value

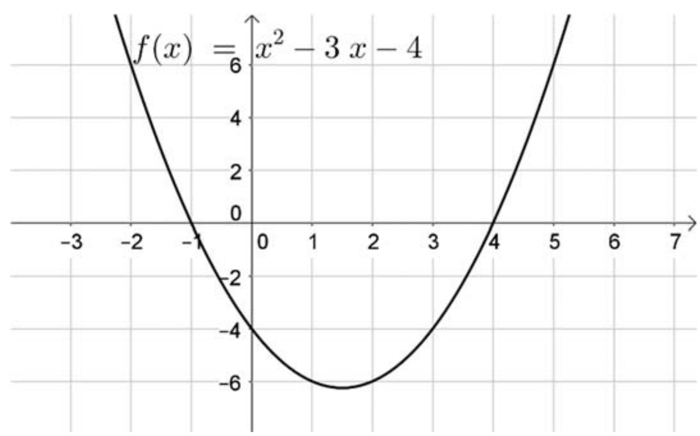
Example 12: $\int_{-1}^{2} |x| \, dx$

Recall that $y = |x|$ *is a piecewise function!*

$$f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$



Example 13: Set-up the integral needed to integrate $\int_{1}^{5} \left| x^2 - 3x - 4 \right| \, dx$.



Try this one: $\int_1^5 \left| x^2 + 4 \right| \, dx$