Section 6.4: Integration by Substitution

The **method of substitution** is based on the Chain Rule: $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$.

When integrating a composite function, we let the variable $u = g(x)$ (or the "inside function") and

and "$g'(x)dx$" by du. This is called **substitution (**or $u -$ **substitution**). All x's in the integrand must be changed to be in terms of u including dx.

Steps:

- Let u = the "inside function".
- Find the derivative of u (don't forget the du).
- Substitute.
- Integrate.

Example 1: $\int 2x\left(x^2+5\right)^3 dx$

Example 2: $\int \cos^6 x \sin x dx$

Example 3: $\int 4x\sqrt{-x^2+12}\,dx$

Example 4: $\int 5xe^{x^2+1}\,dx$

Example 5: $\int e^x \sec^2\left(e^x\right) dx$

Example 6: $\int \sec^2 x \tan x dx$

Two Important Integrals

$$\int \tan x dx = -\ln|\cos x| + C = \ln|\sec x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

For those integrands with fractions, first check to see if it's one of the following:

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \arcsin x + C \qquad \int \frac{1}{1+x^2}\, dx = \arctan x + C \qquad \int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \operatorname{arc\,sec} x + C$$

If not, then if the top function is the derivative of the bottom (or part of it) then let
u = denominator.

Example 7: $\int \frac{1}{\sqrt{1-x^2}} dx$

Example 8: $\int \frac{1}{1+x^2} dx$

Example 9: $\int \frac{2x}{x^2+5} dx$

Example 10: $\int \frac{x}{\sqrt{1-x^2}}\,dx$

The integral of the inverse function with *u*-substitution.

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C \qquad \int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\left(\frac{u}{a}\right)+C$$

Example 11: $\int \frac{1}{9+25x^2}\,dx$

Example 12: $\int \frac{e^x}{\sqrt{1-16e^{2x}}}\,dx$

The method of u-substitution with Definite Integrals

Example 13: $\displaystyle\int_1^2 \frac{2x+1}{x^2+x}\, dx$

Example 14: $\displaystyle\int_{-6}^0 (x+4)^3\, dx$