Course Information
Math 1431
Section 12201
MW 5:30 – 7:00 pm SW 102

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208 PGH

Read your Syllabus!!

Labs

Attendance is mandatory and part of your grade! Homework will be due in lab on the posted dates. You will also have lab quizzes starting this week in lab.

My Math 1431 Labs:

<table>
<thead>
<tr>
<th>Section</th>
<th>Time</th>
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<tr>
<td>12270-13</td>
<td>4:00 – 5:30 PM</td>
<td>C 105</td>
<td>Yingxue Su</td>
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<tr>
<td>12271-14</td>
<td>7:00 – 8:30 PM</td>
<td>SEC 202</td>
<td>Stephen Thacker</td>
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<td>12202-16</td>
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<td>SEC 201</td>
<td>Nickolas Fularczyk</td>
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<td>12203-17</td>
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<td>Xiaoqian Chen</td>
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<td>12204-18</td>
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<td>Nickolas Fularczyk</td>
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CourseWare Accounts

http://www.casa.uh.edu

The first portion of these materials is freely available for the first few days of class. All students must purchase a Course Access Code by the stated date to continue accessing the course learning materials.
Daily Poppers

Daily quizzes (poppers) will be given in lecture starting the third week of class. You will need to purchase a course pack of custom bubbling forms from the bookstore.

NOTE: Make sure you get the correct packet. They are sold by Section Number. If you don’t have the correct section, you will not receive credit. This class is section 12201

Online Quizzes

You may take them up to 20 times each. The highest score is recorded.

Watch for when they are to be closed, and don’t wait until the last day (or minute) to complete them. The system may become overloaded and thus may prevent you from receiving credit.

There is NO AMNESTY at the end of the semester.

Attendance and Classroom Behavior

- Come to class on time.
- Be prepared to start on time.
- Turn off your cell phone.
- Do not read the newspaper, surf the web, or do anything that might disturb other students (including non-calculus discussions).
- Pay attention.
- Ask and answer questions.
- If you must come in late, or leave early, please be respectful of everyone else.
Finding a limit amounts to answering the following question: What is happening to the y-value of a function as the x-value approaches a specific target number? If the y-value is approaching a specific number, then we can state the limit of the function as x gets close to the target number.

Example 1: Given the function, \( f(x) \).

What value do we expect from the left of \( x = -2 \)?
What value do we expect from the right of \( x = -2 \)?
What value do we expect at \( x = -2 \)?
What value do we expect from the left of \( x = 2 \)?
What value do we expect from the right of \( x = 2 \)?
What value do we expect at \( x = 2 \)?

Informal definition of a Limit: We say that the function \( f \) has the limit \( L \) as \( x \) approaches a number \( c \), written \( \lim_{{x \to c}} f(x) = L \) if the value \( f(x) \) can be made as close to the number \( L \) as we like by getting \( x \) sufficiently close to, but not equal to, \( c \).

Note the \( L \) must be a real value, otherwise the limit fails to exist (DNE = Does Not Exist)
Example 2: Let’s revisit the graph from example 1. Evaluate the following limits.

a. \( \lim_{x \to -1} f(x) = 0 \)

b. \( \lim_{x \to 2} f(x) = \text{DNE} \)

c. \( \lim_{x \to 3} f(x) = 2 \)

Notice that when we look for the limit of a function as we approach a specific \( x \)-value, we look at the left and right hand side of the graph. If we are only interested in the behavior of a function when we look from one side and not from the other, we are looking at a **one-sided limit**.

### Left-Hand Limit

\[ \lim_{x \to c^-} f(x) = L \]

### Right-Hand Limit

\[ \lim_{x \to c^+} f(x) = L \]

Which bring us to a more formalized definition.

Let \( f \) be a function that is defined for all values of \( x \) close to \( c \), except perhaps at \( c \) itself. Then \( \lim_{x \to c} f(x) = L \) if and only if \( \lim_{x \to c^-} f(x) = L \) and \( \lim_{x \to c^+} f(x) = L \).

Example 3: Given the graph of the function \( f \), evaluate the following if possible.

\[
\begin{align*}
 f(0) &= \text{DNE} & \lim_{x \to 0^-} f(x) &= 0 & \lim_{x \to 0^+} f(x) &= 0 & \lim_{x \to 0} f(x) &= 0 \\
 f(1) &= \text{DNE} & \lim_{x \to 1^-} f(x) &= 3 & \lim_{x \to 1^+} f(x) &= +\infty & \lim_{x \to 1} f(x) &= \text{DNE}
\end{align*}
\]
Math1431 Section 1.2

\[ f(2) = -2 \]

\[ \lim_{x \to 2^-} f(x) \]

\[ \lim_{x \to 2^+} f(x) = -1 \]

\[ \lim_{x \to 2} f(x) = DNE \]

Example 4: Evaluate each limit, if it exist. (Remembering their graphs could help)

a. \[ \lim_{x \to 3} (x^2 - 1) = 8 \]

b. \[ \lim_{x \to -4^+} \sqrt{x + 4} = 0 \]

c. \[ \lim_{x \to -4^-} \sqrt{x + 4} = DNE \]

d. \[ \lim_{x \to 0^+} \frac{|x|}{x} = 0 \]

\[ \begin{cases} \frac{-x}{x} = -1 & x < 0 \\ \frac{x}{x} = 1 & x > 0 \end{cases} \]

\[ |x| = 0 \]

\[ x < 0 \]

\[ x = 0 \]

\[ x > 0 \]

\[ = DNE \]

e. \[ f(x) = \begin{cases} x^2, & x < 2 \\ 3x, & x \geq 2 \end{cases} \]

\[ \lim_{x \to 2^-} f(x) = DNE \]

\[ \lim_{x \to 2^+} f(x) \]

\[ = 4 \]

\[ \lim_{x \to 2^+} f(x) = 6 \]
Section 1.3: The Definition of a Limit

The Limit of a Function

If we let the limit of a function be equal to $L$ and $c$ be the fixed value that $x$ approaches, then we can say
\[ \lim_{x \to c} f(x) = L \text{ if and only if for each } \varepsilon > 0, \text{ there exists a } \delta > 0 \text{ such that if } |x - c| < \delta, \text{ then } |f(x) - L| < \varepsilon. \]

Example 1: Show that \( \lim_{x \to 2} (5x - 2) = 8 \) using the definition of a limit.

Identify the following pieces
\[ c = \quad f(x) = \quad L = \]
Math1431  Section 1.3

Example 2: Give the largest \( \delta \) that works with \( \varepsilon = 0.1 \) for the limit, \( \lim_{x \to -1} (1 - 2x) = 3 \)

Arithmetic Rules for Limits

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \) then:

1. \( \lim_{x \to c} [f(x) \pm g(x)] = L \pm M \)
2. \( \lim_{x \to c} [k \cdot f(x)] = kL \)
3. \( \lim_{x \to c} [f(x) \cdot g(x)] = LM \)
4. \( \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}, \quad M \neq 0 \)

Example 3: Let \( \lim_{x \to c} f(x) = 3 \), \( \lim_{x \to c} g(x) = -2 \) and \( \lim_{x \to c} h(x) = -5 \). Evaluate the following

\( \lim_{x \to c} (2f(x) - g(x) \cdot h(x) + [h(x)]) \)

Example 4: Evaluate the following limits:

a. \( \lim_{x \to 2} \frac{3x^2 + 2x}{2x + 1} \)
b. \( \lim_{x \to -1} 2x^3 - 3x^2 + 4x + 12 \)

c. \( \lim_{x \to -1} \frac{3x^2 + 2x}{2x^2 + 3x + 1} \)

d. \( \lim_{x \to 1} \frac{2x^3 - 2x}{x - 1} \)

So if a simple plug in didn’t give us an actual answer, which means we have to approach the limit a different way.

1. Factoring
2. Distribution, common denominators
3. Using the conjugate

**Example 5:** Back to 4d

\[ \lim_{x \to 1} \frac{2x^3 - 2x}{x - 1} \]
Example 6: Evaluate: \( \lim_{{x \to 0}} x \left( 4 - \frac{7}{x} \right) \)

Example 7: Evaluate: \( \lim_{{x \to 3}} \frac{2x^3 + 54}{x + 3} \)

Example 8: Evaluate \( \lim_{{x \to 25}} \frac{x - 25}{\sqrt{x} - 5} \)
Example 9: Evaluate \( \lim_{x \to -1} \frac{|x + 1|}{x + 1} \)

\[
\frac{1}{4} \cdot \frac{1}{h + 4} \quad \text{for } h \to 0
\]

Example 10: Evaluate \( \lim_{h \to 0} \frac{4}{h + 4} \)

Limit of Piecewise Functions

Example 11: Let \( f(x) = \begin{cases} 
\sqrt{5-x}, & x < 0 \\
x^2, & 0 \leq x < 2 \\
-4x + 16, & x \geq 2 
\end{cases} \), find \( \lim_{x \to 0} f(x) \)
Example 12: \( f(x) = \begin{cases} \frac{x^2 - 16}{x - 4}, & x \neq 4 \\ 9, & x = 4 \end{cases} \), find \( \lim_{x \to 4} f(x) \)

Limits at Infinity

If a limit at infinity exists and it’s equal to a single real number \( L \) then they are written as \( \lim_{x \to \infty} f(x) = L \) or \( \lim_{x \to -\infty} f(x) = L \). These limits deal with what is happening to the \( y \)-values to the far left or right side of the graph (function).

Limits at infinity problems often involve rational expressions (fractions). The technique we can use to evaluate limits at infinity is to recall some rules from Algebra used to find horizontal asymptotes. These rules came from “limits at infinity” so they’ll surely work for us here.

The highest power of the variable in a polynomial is called the **degree** of the polynomial.

We can compare the degree of the numerator with the degree of the denominator and come up with some generalizations.

- If the degree of the numerator is smaller than the degree of the denominator, then \( \lim_{x \to \infty} \frac{f(x)}{g(x)} = 0 \)

- If the degree of the numerator is the same as the degree of the denominator, then you can find \( \lim_{x \to \infty} \frac{f(x)}{g(x)} \) by making a fraction from the leading coefficients of the numerator and denominator and then reducing to lowest terms.

- If the degree of the numerator is larger than the degree of the denominator, then the limit does not exist.
Let's see how these generalizations came to be.

**Example 13:** Evaluate the following limits

\[
\lim_{x \to \infty} \frac{1}{x}
\]

\[
\lim_{x \to \infty} \frac{1}{x^2}
\]

\[
\lim_{x \to \infty} \frac{1}{x^n}
\]

**Example 14:** Evaluate \( \lim_{x \to \infty} \frac{2x + 7}{x^4 + 5x^2 + 6} \)
Example 15: Evaluate \( \lim_{x \to \infty} \frac{2x^2 - 3x + 1}{4x - x^2} \)