Evaluate the following:

21. \( \lim_{{x \to -3}} f(x) \)  
   \( x \to -3 \)

22. \( f(-5) \)

27. \( \lim_{{x \to 5^-}} f(x) \)  
   \( x \to 5^- \)

28. \( \lim_{{x \to 2^-}} f(x) \)  
   \( x \to 2^- \)

A.  1  B.  -2  C.  0  
D.  -1  E.  DNE

Question 10: Evaluate \( \lim_{{x \to \infty}} \frac{x^2 + 2x - 8}{x^2 + x - 6} \)

A. 4  B. 5  C. 7  D. 0  E. DNE

Choose E  B  D  0  D

# 11 is B

# 50 is A
Theorem: Let \( k \) be any real number. If \( f \) and \( g \) are differentiable at \( x \), then so are \( f + g \) and \( f - g \). Moreover, 
\[
(f \pm g)'(x) = f'(x) \pm g'(x)
\]

Example 4: Find the derivative: 
\[
f(x) = -\frac{3}{4} x^6 + \frac{5}{3} + 6\sqrt[3]{x^5} + 13
\]
\[
f'(x) = -\frac{3}{4} \cdot 6x^5 + 5 \cdot \frac{5}{3} x^{-\frac{4}{3}} + 6 \cdot \frac{1}{3} x^{\frac{2}{3}} + 0
\]
\[
= -\frac{9}{2} x^5 - \frac{15}{x^{\frac{4}{3}}} + \frac{4}{3\sqrt[3]{x}}
\]

Example 5: Find the derivative: 
\[
f(x) = \frac{3x^5 - 7x^2 + 3}{x^3}
\]

Rewrite
\[
f'(x) = 3x^2 - \frac{7}{x} + \frac{3}{x^3} = 3x^2 - 7x^{-1} + 3x^{-3}
\]
\[
f'(x) = 3 \cdot 2x^{1-1} - 7(-1)x^{-2} + 3(-3)x^{-4}
\]
\[
= 6x + \frac{7}{x^2} - \frac{9}{x^4}
\]
In some cases, we may want to find all points for which the tangent line to the graph of a function is horizontal or equal to a specified number. Set the derivative equal to the given number and solve for \( x \).

**Example 6:** Find all \( x \)-value(s) on the graph of \( f(x) = x^3 + 5x^2 \) where the tangent line is horizontal.

\[
\frac{df}{dx}(x) = 3x^2 + 10x
\]

\[
3x^2 + 10x = 0
\]

\[
x(3x + 10) = 0
\]

\[
x = 0, \quad -\frac{10}{3}
\]

**Example 7:** Find all \( x \)-value(s) on the graph of \( f(x) = x^2 + 4x + 1 \) where \( f'(x) = 5 \).

\[
\frac{df}{dx}(x) = 2x + 4
\]

\[
2x + 4 = 5
\]

\[
x = \frac{1}{2}
\]

**Tangent and Normal Lines**

We already known what a tangent line is all about.

A **normal line to a curve** at a particular point is the line through that point and perpendicular to the tangent.

Recall from Pre-Algebra the slope of any line perpendicular to a line with slope \( m \) is the negative reciprocal.

\[
m_1 \cdot m_2 = -1
\]
Example 8: Find the equations of the tangent and normal line to \( f(x) = 3x^2 + 4x + 2 \) at \( x = -1 \)

\[
\begin{align*}
f'(x) &= 6x + 4 \\
\frac{d}{dx} f(x) &= 6(-1) + 4 \\
&= -2 \quad \text{Slope of Tangent}
\end{align*}
\]

Tangent

\[
\begin{align*}
y - y_1 &= m(x - x_1) \\
y - 1 &= -2(x + 1)
\end{align*}
\]

Normal Line

\[
\begin{align*}
y - 1 &= \frac{1}{2}(x + 1)
\end{align*}
\]

Higher Order Derivatives

\[
\begin{align*}
&f'(x), \quad f''(x), \quad f'''(x), \quad f^{(4)}(x) \\
&\frac{d}{dx} f(x), \quad \frac{d^2}{dx^2} f(x), \quad \frac{d^3}{dx^3} f(x), \quad \frac{d^4}{dx^4} f(x)
\end{align*}
\]

Example 9: Find \( \frac{d^3}{dx^3} f(x) \) for \( f(x) = 2x^5 - 3x^3 + 7x \)

\[
\begin{align*}
f'(x) &= 10x^4 - 9x^2 + 7 \\
f''(x) &= 40x^3 - 18x \\
f'''(x) &= 120x^2 - 18
\end{align*}
\]

Derivatives of the Trigonometric Functions

\[
\begin{align*}
(sin x)' &= \cos x \\
(cos x)' &= -\sin x \\
(tan x)' &= \sec^2 x \\
(csc x)' &= -\csc x \cot x \\
(see x)' &= \sec x \tan x \\
(cot x)' &= -\csc^2 x
\end{align*}
\]
Example 10: Find the slope of the tangent line to the function \( f(x) = 4\tan x - 6\cos x \) at \( x = \frac{\pi}{4} \).

\[
\begin{align*}
    f'(x) &= 4\sec^2 x - 6\cdot(-\sin x) \\
    &= 4\sec^2 x + 6\sin x \\
    f'(\frac{\pi}{4}) &= 4\sec^2\left(\frac{\pi}{4}\right) + 6\sin\left(\frac{\pi}{4}\right) \\
    &= 4\cdot2 + 6\cdot\frac{\sqrt{2}}{2} \\
    &= 8 + 3\sqrt{2}
\end{align*}
\]

Example 11: Find all values of \( x \) on \( [0, 2\pi) \) where the tangent line is horizontal to \( f(x) = 6\sqrt{3}\sin x + 18\cos x \).

\[
\begin{align*}
    f'(x) &= 6\sqrt{3}\cos x + 18\cdot(-\sin x) \\
    &= 6\sqrt{3}\cos x - 18\sin x \\
    6\sqrt{3}\cos x - 18\sin x &= 0 \\
    6\sqrt{3}\cos x &= 18\sin x \\
    \frac{\sqrt{3}}{3} &= \tan x \\
    x &= \frac{\pi}{6}, \frac{7\pi}{6}
\end{align*}
\]

Question 18: Evaluate \( f(x) = 6\sin x + 10\cos x \) at \( x = \frac{-\pi}{4} \).

a. \( 2\sqrt{2} \)
b. \( 8\sqrt{2} \)
c. \( -2\sqrt{2} \)
d. \( 4\sqrt{2} \)
e. None of the above
Math1431 Section 2.2

Now that we know much easier rules for finding derivatives, let’s revisit differentiability with piecewise functions.

**Example 12:** Is this function differentiable?

\[
f(x) = \begin{cases} 
3x & x > 3 \\
3x + 1 & x \leq 3 
\end{cases}
\]

\[
f'(x) = \begin{cases} 
3 & x > 3 \\
2x + 1 & x \leq 3 
\end{cases}
\]

**Question 17:** Which function do you need to find the slope of tangent lines?

- a. 2nd Derivative
- b. 3rd Derivative
- c. 1st Derivative
- d. Original function
- e. None of these
Question 14: Evaluate

\[
\lim_{x \to -\infty} \frac{6 - 2x}{\sqrt{9x^2 + 1}}
\]

A. 2/9  
B. -2/9  
C. 2/3  
D. -2/3  
E. DNE
Math 1431 – Fall 2018

Student should review:

1. All quizzes 0 – 6
2. All homeworks 1 – 3
3. All notes from 1.1 – 1.6 and 2.1
4. Practice test 1
5. At least these review videos found under math placement: 4abcdf, 5abcd, 7abcdef

Test Topics:

Student should be able to:

1. Determine the domain of any of the following type of function [see video section 4c]:
   a. Polynomial [see video section 4c]
   b. Rational [note: be able to distinguish between a hole and a VA] -- [see video section 5d]
   c. Radical [see video section 4c and 4f]
2. Determine the inverse of a function [see video section 4f]
3. Find the horizontal asymptote of a rational function. [see video section 5d]
4. Be able to find exact values for:
   a. Inverse trig functions [see video section 7d]
   b. Trig functions [see video section 7b]
5. Be able to solve a simple trig equations [see video section 7f]
6. Be able to find limits using:
   a. A graph
   b. A table of values
   c. Algebraic techniques
   d. A special limit, such as \( \lim_{x \to 0} \frac{\sin(x)}{x} = 1 \)
7. Be able to recite and use the \( \epsilon - \delta \) limit definition
8. Be able to find values for A and B so a piecewise function is continuous
9. Be able to recite and use the IVT
10. Be able to recite and use the limit definition of a derivative
11. Be able to recite and use the definition of continuity at \( x = c \)
12. Be able to recite definition for and identify types of discontinuities

Test is 75 minutes, has 19 questions. 4 FR are 10 points each and the 15 MC are 4 points each.