Example 1: Pull Company installed a new machine in one of its factories at a cost of $150,000. The machine is depreciated linearly over 10 years with no scrap value. Find an expression for the machine’s book value in the t-th year of use (0 ≤ t ≤ 10).

\[
\begin{align*}
M &= \frac{\text{Cost} - \text{Salvage Value}}{\text{Useful Life}} \\
&= \frac{150000 - 0}{10} \\
&= 15000 \\

d = x = 0 \\
V(t) &= mt + \text{Initial} \\
&= 15000t + 1500000
\end{align*}
\]

Example 2: A piece of equipment was purchased by a company for $10,000 and is assumed to have a scrap value of $3,000 in 5 years. If its value is depreciated linearly, find the value of the equipment after 3 years (0 ≤ t ≤ 5).

\[
\begin{align*}
M &= \frac{\text{Cost} - \text{Salvage Value}}{\text{Useful Life}} \\
&= \frac{10000 - 3000}{5} \\
&= 1400 \\

d = x = 0 \\
V(t) &= mt + \text{Initial} \\
&= -1400t + 10000 \\
V(3) &= -1400(3) + 10000 = 5600
\end{align*}
\]

Example 3: A bicycle manufacturer experiences fixed monthly costs of $75,000 and fixed costs of $75 per standard model bicycle produced. The bicycles sell for $125 each.

a. What is the cost, revenue and profit functions?

\[
\begin{align*}
C(x) &= 75x + 75000 \\
R(x) &= 125x \\
P(x) &= R(x) - C(x) = 125x - (75x + 75000) \\
&= 50x - 75000
\end{align*}
\]

b. What is the break-even point?

\[
\begin{align*}
R(x) &= C(x) \\
125x &= 75x + 75000 \\
50x &= 75000 \\
x &= 1500 \\
\text{Break Even Quantity} &= 1500 \\
\text{Break Even Revenue} &= R(1500) = 125(1500) = 187500
\end{align*}
\]
Example 4: Solve using Gauss-Jordon.

\[
\begin{bmatrix}
1 & 3 & 1 & | & 3 \\
0 & 1 & 0 & | & 2 \\
1 & -6 & 0 & | & -13
\end{bmatrix}
\quad \rightarrow
\begin{bmatrix}
1 & 3 & 1 & | & 3 \\
0 & -3 & -1 & | & -3 \\
0 & -9 & -1 & | & -13
\end{bmatrix}
\quad \rightarrow
\begin{bmatrix}
1 & 3 & 1 & | & 3 \\
0 & 1 & 0 & | & 2 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 3 & 1 & | & 3 \\
0 & -3 & -1 & | & -3 \\
0 & -9 & -1 & | & -13
\end{bmatrix}
\quad \rightarrow
\begin{bmatrix}
1 & 3 & 1 & | & 3 \\
0 & 1 & 0 & | & 2 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\quad \rightarrow
\begin{bmatrix}
1 & 0 & 1 & | & -3 \\
0 & 1 & 0 & | & 2 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\quad \rightarrow
\begin{bmatrix}
1 & 0 & 1 & | & -3 \\
0 & 1 & 0 & | & 2 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\quad \rightarrow
\begin{bmatrix}
1 & 0 & 1 & | & 0 \\
0 & 1 & 0 & | & -2 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\]

\[
X = 1 \\
Y = 2 \\
Z = -2
\]

Example 5: Determine which of the following matrices are in row-reduced form. If a matrix is not in row-reduced form, state why.

\(a. \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}\) 
\(b. \begin{bmatrix} 0 & 2 & 0 \end{bmatrix}\) 
\(c. \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}\)

Yes 

No, 2's in wrong direction 

Yes, 9 is not an issue
**Example 6:** The reduced form for the augmented matrix of a system with 3 equations and 3 unknowns is given. Give the solution to the system, if it exists.

a. \[
\begin{bmatrix}
1 & 0 & -5 \\ 0 & 1 & 0 \\ 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
-3 \\ -1 \\ 6
\end{bmatrix}
\] \[0 = 6\] 
Not True

No Solution

b. \[
\begin{bmatrix}
1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
0 \\ 1 \\ 0
\end{bmatrix}
\] \[x = 0\] \[y = 1\] \[z = 3\]

c. \[
\begin{bmatrix}
1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 \\ 2 \\ 3
\end{bmatrix}
\] \[x + z = 1\] \[y = 2\] 
Variables > Equations 
No Solution or Infinite

**Example 7:** Find the value for x and y:

\[
\begin{bmatrix}
1 & 2 & 3 \\ 3 & 4 & 1 \\ -1 & -3 & 4
\end{bmatrix}
= \begin{bmatrix}
-1 \\ 2 \\ 4
\end{bmatrix}
\]

\[
x - 3(y - 1) = 2(-4)
\]
\[x - 12 = -6\]
\[x = 20\]

\[
y - 3y + 2 = -8
\]
\[-3y = -12\]
\[y = 4\]

**Example 8:** Given the following matrices find the product.

\[
\begin{bmatrix}
0 & -2 & 1 \\ 4 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\ 0 & 1 \\ -2 & -1
\end{bmatrix}
\] \[2 \times 2\]

\[
\begin{align*}
R_1 \cdot C_1 &= 0(1) - 2(0) + 1(-2) \\
&= 0 - 0 - 2 \\
R_2 \cdot C_1 &= 4(1) - 1(0) + 0(-2) \\
&= 4 - 0 + 0
\end{align*}
\]

\[
\begin{align*}
R_1 \cdot C_2 &= 0(-3) - 2(1) + 1(-1) \\
&= 0 - 2 - 1 \\
R_2 \cdot C_2 &= 4(-3) - 1(1) + 0(-1) \\
&= -12 - 1 + 0
\end{align*}
\]

\[
= \left(\begin{array}{cc}
-2 & -3 \\
4 & -9
\end{array}\right)
\]
Midterm Review

Example 9: Find the transpose of matrix A.

$$A = \begin{bmatrix} 1 & -4 & 3 \\ -2 & 7 & 4 \\ 3 & 4 & 3 \end{bmatrix}$$

$$A^\top = \begin{pmatrix} 1 & -2 \\ -4 & 7 \\ 3 & 4/3 \end{pmatrix} = 3 \times 2$$

Example 10: Find the inverse of matrix A.

$$\begin{bmatrix} -3 & 4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$D = ad - bc$$
$$= -3(-2) - 4(1)$$
$$= 6 - 4$$
$$= 2$$

$$A^{-1} = \frac{1}{D} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -2 & -4 \\ -1 & -3 \end{bmatrix}$$

$$= \begin{pmatrix} -1 & -2 \\ -1/2 & -3/2 \end{pmatrix}$$

Example 11: Solve the system of equations by using the inverse of the coefficient matrix.

$$x - y = -4$$
$$5x + 6y = 2$$

$$A = \begin{pmatrix} 1 & -1 \\ 5 & 6 \end{pmatrix}$$

$$X = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$B = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$D = ad - bc$$
$$= 6(6) - (-1)(5)$$
$$= 36 + 5$$
$$= 41$$

$$A^{-1} = \frac{1}{D} \begin{bmatrix} 6 & 1 \\ -5 & 1 \end{bmatrix}$$

$$= \frac{1}{41} \begin{pmatrix} 6 & 1 \\ -5 & 1 \end{pmatrix}$$

$$X = A^{-1} \cdot B$$

$$= \frac{1}{41} \begin{pmatrix} 6 & 1 \\ -5 & 1 \end{pmatrix} \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$= \frac{1}{41} \begin{pmatrix} -24 + 2 \\ 20 + 2 \end{pmatrix}$$

$$= \frac{1}{41} \begin{pmatrix} -22 \\ 22 \end{pmatrix}$$

$$x = -2$$
$$y = 2$$

($$-2, 2$$)
**Example 12:** A vineyard produces two special wines a white and a red. A bottle of the white wine requires 14 pounds of grapes and one hour of processing time. A bottle of red wine requires 25 pounds of grapes and 2 hours of processing time. The vineyard has on hand 2,198 pounds of grapes and can allot 160 hours of processing time to the production of these wines. A bottle of the white wine makes $11.00 profit, while a bottle of the red wines makes $20.00 profit. Set-up the linear programming problem so that profit can be maximized.

\[
\begin{align*}
\text{Maximize} & \quad P = 11x + 20y \\
\text{Subject to} & \quad 14x + 25y \leq 2198 \\
& \quad x + 2y \leq 160 \\
& \quad x, y \geq 0
\end{align*}
\]

**Example 13:** Solve the linear programming problem.

Max \( P(x) = 3x + 7y \)

St: 
1. \( 2x + 5y \leq 20 \) 
2. \( x + y \leq 7 \) 
3. \( x, y \geq 0 \)

Graphically, the feasible region is found by plotting the lines and identifying the corner points. The feasible region is a polygon bounded by the lines \( 2x + 5y = 20 \), \( x + y = 7 \), \( x = 0 \), and \( y = 0 \). The corner points are found by solving the equations of the lines that form the boundary.

- Intersection of \( 2x + 5y = 20 \) and \( x = 0 \): \( (0, 4) \)
- Intersection of \( x + y = 7 \) and \( y = 0 \): \( (7, 0) \)
- Intersection of \( x = 0 \) and \( y = 0 \): \( (0, 0) \)
- Intersection of \( x + y = 7 \) and \( 2x + 5y = 20 \) (solving the system of equations):
  - \( x + y = 7 \)
  - \( 2x + 5y = 20 \)
  - Solve to get \( x = 3, y = 4 \)

Evaluating the objective function at these corner points:

- \( P(0, 4) = 3(0) + 7(4) = 28 \)
- \( P(7, 0) = 3(7) + 7(0) = 21 \)
- \( P(3, 4) = 3(3) + 7(4) = 39 \)
- \( P(0, 0) = 3(0) + 7(0) = 0 \)

The maximum value of the objective function is \( 39 \) at the point \( (3, 4) \), which is the optimal solution.
Midterm Review

**Example 14:** Solve the linear programming problem.

Min \( C(x) = x + 6y \)

St: 
- \( 3x + 4y \geq 36 \)
- \( 2x + y \geq 14 \)
- \( x, y \geq 0 \)

**Line 1**
- \( x \)-int: \((12,0)\)
- \( y \)-int: \((0,9)\)

\[ 3x + 4y \geq 36 \]

\[ y \geq -\frac{3}{4}x + 9 \]

\[ \frac{5}{4}x = 5 \]

\[ x = 4 \]

\[ y = 6 \]

**Line 2**
- \( x \)-int: \((7,2)\)
- \( y \)-int: \((0,14)\)

\[ 2x + y \geq 14 \]

\[ y \geq -2x + 14 \]

\[ \frac{3}{4}x + 9 = -2x + 14 \]

\[ x = 4 \]

\[ y = 6 \]

For examples 15 – 19, state the type of problem and calculate the answer.

15. Parents of a college student wish to set up an account that will pay $350 per month to the student for four years. How much should they deposit now at 9% annual interest, compounded monthly?

**Annuity**

\[ P = E \left[ \frac{1 - (1 + \frac{r}{t})^t}{\frac{r}{t}} \right] = E \left( 1 - (1 + \frac{r}{t})^{-n} \right) \]

\[ C = \frac{r}{t} \]

\[ n = mt \]

\[ P = 14,060 \]
16. Your friend’s payments on his new car are $524.37 per month. He received a $3000 trade-in on his old car, and received a financing package that was 8.9% annual interest, compounded monthly for five years. What was the total purchase price of the car?

\[ P = E \left[ 1 - \left(1 + \frac{i}{12}\right)^{-5\times12} \right] \]

\[ P = 28,319.83 + 3000 = 28,319.83 \]

17. A company estimates that it will have to replace a piece of equipment at a cost of $10,000 in 5 years. The owner wants to have this money available when the equipment is replaced. He can make fixed quarterly payments and earn interest at 6% annual interest compounded quarterly. How much should the payments be?

\[ E = \frac{P \times i}{(1+i)^n - 1} = \frac{E \times i}{(1+i)^n - 1} \]

\[ = 432.40 \]

18. Jenna wants to begin saving money for a new car. She can make monthly payments of $150 into an account at her credit union which pays 5% annual interest compounded monthly. How much money will she have available for her new car in three years?

\[ F = E \left( 1 + \frac{i}{12} \right)^{3\times12} \]

\[ = 55,913 \]

19. The manager of a manufacturing company knows that they will need a new machine in one of their factories. The new machine will cost them $12,500. The manager has determined that they can afford to pay 20% of the cost of the machine in cash. They can then finance the rest through a credit union. The credit union will charge 2% per year compounded monthly. How much are their monthly payments for 4 years?

\[ E = \frac{P \times i}{1 - (1+i)^n} \]

\[ P = 12500 - 20\% = 12500 - 0.20 \times 12500 = 10000 \]

\[ E = 216.95 \]