Math 1313  Section 4.3

Section 4.3 - Amortization and Sinking Funds

Sinking Fund

Is a fund accumulated over time in order to pay off a debt or meet future goals or obligations.

The periodic payment $E$ required to accumulate a sum of $F$ dollars over $n$ periods with interest charged at the rate of $i$ per period is

$$ E = \frac{Fi}{(1+i)^n - 1} $$

Amortization

Is the process of paying off a debt with equal periodic payments made over a specified period of time that includes a portion of the principal and interest.

The periodic payment $E$ on a loan of $P$ dollars to be amortized over $n$ periods with interest charged at the rate of $i$ per period is

$$ E = \frac{Pi}{1-(1+i)^{-n}} $$

Example 1:  Kelly wishes to buy a car that costs $32,998. The car dealer tells her that they can finance the car at 6.25% per year compounded monthly for 5 years. She decides to secure the loan from the dealer. How much will her monthly payments be?

$$ E = \frac{P \cdot i}{1-(1+i)^{-n}} = \frac{32,998 \cdot 0.0625/12}{1-(1+0.0625/12)^{-60}} $$

$$ E \approx 5641.79 $$
Example 2: A person would like to have $200,000 in an account for retirement 15 years from now. How much should be deposited quarterly in an account paying 6% per year compounded quarterly to obtain this amount?

\[
E = \frac{F \cdot i}{(1+i)^n - 1} = \frac{200000 \cdot 0.06/4}{(1+0.06/4)^{60} - 1}
\]

\[
= \frac{200000 \cdot 0.06/4}{(1+0.06/4)^{60} - 1}
\]

\[
= 2,076.69
\]

Example 3: A sailboat costs $16,000. You pay 15% down and secure a loan for the remaining balance. How much are your monthly payments if 18% per year compounded monthly is charged over a period of 6 years?

\[
P = 16000 - 15\%
\]

\[
P = 16000 - 0.15(16000)
\]

\[
P = 13,600
\]

\[
r = 0.18
\]

\[
\frac{r}{12} = \frac{0.18}{12}
\]

\[
t = 6
\]

\[
m = 12
\]

\[
E = P \cdot i \left( 1 - (1+i)^{nt} \right)
\]

\[
= 13600 \cdot 0.18/12 \left( 1 - (1+0.18/12)^{-72} \right)
\]

\[
= 310.19
\]

Example 4: Christina plans to go to Disney World in two summers and wishes to have $7000 by then. How much money should she deposit monthly for the next 2 years in an account paying 3.25% per year compounded monthly to achieve this goal?

\[
E = F \cdot i \left( 1 + i \right)^{nt} - 1
\]

\[
= 7000 \cdot 0.0325/12 \left( 1 + 0.0325/12 \right)^{24} - 1
\]

\[
= 282.69
\]
Example 5: Business partners, Bill and Bob, buy an apartment house for $1,250,000 by making a down payment of $125,000 and financing the rest with semiannual payments over the next 10 years. The interest rate on the debt is 8% per year compounded semiannually. How much is their semiannually payment?

\[
E = \frac{P \cdot \frac{r}{2}}{1 - (1 + \frac{r}{2})^{-2n}}
\]

\[
P = 1,250,000 - 125,000 = 1,125,000
\]

\[
t = 10
\]

\[
r = 0.08
\]

\[
m = 2
\]

\[
\frac{r}{2} = 0.04
\]

\[
E = \frac{1,125,000 \times 0.04}{1 - (1 + 0.04)^{-20}} = 9,672,779.47
\]